Cash Mergers and the Volatility Smile

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Abstract

In an empirical study of cash mergers since 1996, we find that the equity options on target firms display a pronounced smile pattern in their implied volatilities which gets more pronounced when the merger success probability gets higher. We propose an arbitrage-free model to analyze option prices for firms undergoing a cash merger attempt. Our formula matches well the observed merger volatility smile. Furthermore, as predicted by the model, we show empirically that the merger volatility smile has a kink at the offer price, and that the magnitude of the kink is proportional to the merger success probability.

JEL Classification: G13, G34, C58.

Keywords: Mergers and acquisitions, Black–Scholes formula, success probability, fallback price, Markov Chain Monte Carlo.

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1 Introduction

One of the most common violations of the Black and Scholes (1973) formula is the volatility smile (or smirk, or sneer), which is a pattern whereby at-the-money equity options have lower implied volatilities than either in-the-money or out-of-the-money options.\(^1\) In the case of European options on S&P 500 futures, Rubinstein (1994) and Jackwerth and Rubinstein (1996) show that the volatility smile has become economically significant only after the 1987 market crash, period for which they also record strong violations of the assumption of a log-normal distribution for the underlying equity prices. The volatility smile has been documented in many other option markets, e.g., for options on currencies or fixed income securities, and various explanations have been proposed for the volatility smile, including stochastic volatility or jumps in the underlying prices, fear of crashes, etc.\(^2\)

If changes in the probability of a stock index crash can affect the volatility smile, then one would expect certain extreme events in the life of a firm to also affect the volatility smile. A natural candidate for an extreme event is the firm being the target of a merger attempt.\(^3\) Compared with studying market crashes, mergers have the advantage that many of the variables involved in a merger deal are observable, e.g., the offer price, or the effective date (when the merger is expected to be completed). To simplify our analysis, we consider only mergers for which the offer is made entirely in cash.\(^4\)

We then analyze all cash mergers announced between January 1996 and December 2014, and study the effect of the merger on the implied volatility smile for the target firm. Indeed, we find that the options on the target company undergoing a cash merger display a pronounced volatility smile, that we call the merger volatility smile. Furthermore, we show that the shape of the merger volatility smile depends crucially on the probability of

\(^1\)Black and Scholes (1973) assume that the underlying equity price follows a log-normal distribution with constant volatility, which means that the implied volatilities should be the same, irrespective of the strike price of the option.


\(^3\)Black (1989) points out that the Black and Scholes (1973) formula is unlikely to work when the company is the subject of a merger attempt.

\(^4\)When studying options on the target company of a cash merger, as a first approximation we can ignore the stock price of the acquirer, which leads to a simpler model.
Figure 1: Median Implied Volatility Smile for Successful and Failed Mergers. This figure plots Black-Scholes daily implied volatilities ($\sigma_{\text{implied}}$) for the call options on the target company of the 843 cash mergers announced between January 1996 and December 2014 with options traded on the target company. Option prices are either the bid price (‘bid’), ask price (‘ask’), or bid-ask midpoint (‘mid’). On the x-axis is the option moneyness $m$: if $K$ is the strike price of the option and $B_1$ the cash offer price per share, for each $m \in 0.7, 0.75, \ldots, 1.2$ we consider the median implied volatility $\sigma_{\text{implied}}$ corresponding to the call options for which $K/B_1$ rounds to $m$. For each $m$ we plot two median implied volatilities: one computed only for the target companies in deals that eventually succeeded (‘success’), and one computed only for the target companies in deals that eventually failed (‘failure’).

success of the cash merger: the smile is more pronounced when the probability of success is higher. To illustrate this, Figure 1 shows the difference between the median merger volatility smile for cash mergers that eventually succeeded (which on average should have a higher success probability) and the median merger volatility smile for cash mergers that eventually failed (which on average should have a lower success probability). In addition, for the successful deals we notice a pronounced kink in the volatility smile that occurs when the option strike price is close to the target offer price. These findings are
qualitatively the same whether as call option prices we use the end-of-day bid price, ask price, or bid-ask midpoint.

To analyze further the merger volatility smile and understand the source of the kink in Figure 1, we propose a theoretical no-arbitrage model that prices options on the equity of a firm subject to a merger offer. The model is perhaps the simplest extension of the Black and Scholes (1973) model adapted to cash mergers. The model predicts that option prices on the equity of the target firm should exhibit a merger volatility smile, with a kink at the offer price. The size of the kink, i.e., the difference between the tangent slopes at the offer price, should be proportional to the success probability of the merger. An empirical test for cash mergers with options traded on the target company strongly supports the model.

Our model also produces stochastic volatility for the underlying equity price. But, rather than assuming stochastic volatility as an exogenous process, we show that it arises endogenously as a function of the probability of success and the other variables in the structural model. Figure 2 shows the stark difference between the median Black and Scholes (1973) at-the-money implied volatility for cash mergers that eventually succeed versus the median implied volatility for cash mergers that eventually fail. In particular, when then merger is close to a successful completion, the implied volatility is low. This is in line with intuition: when the merger is close to being successful, the equity price is close to the cash offer, thus has low volatility.

More specifically, in our theoretical model we consider a cash merger, in which a company $A$, the acquirer, makes a cash offer to a company $B$, the target. The offer is usually made at a significant premium compared with $B$’s pre-announcement stock price, about 35% in our sample. Therefore, the distribution of the stock price of $B$ is not log-normal, but usually bi-modal: if the deal is successful, the price rises to the offer price, $B_1$; if the deal is unsuccessful, the price reverts to a fallback price, $B_2$, that we assume to have a log-normal distribution.\footnote{The fallback price reflects the value of the target firm $B$ based on fundamentals, but also based on other potential merger offers. The fallback price therefore should not be thought as some kind of fundamental price of company $B$, but simply as the price of firm $B$ if the current deal fails.} We also model the success probability of the deal as a stochastic process, similar to a log-normal process, but constrained to be in $[0,1]$. As in the martingale approach to the Black–Scholes formula, instead of
using the actual success probability, we focus on the risk neutral probability, $q$.\footnote{In the absence of time discounting, the risk neutral probability $q(t)$ would be equal to the price at $t$ of a digital option that offers 1 if the deal is successful and 0 otherwise.} When the success probability, $q$, and the fallback price, $B_2$, are uncorrelated, our formula takes a particularly simple form: the stock price of $B$ is a mixture of $B_1$ and $B_2$ with weights given by $q$ and $1 - q$; and similarly for the prices of European options on $B$ with expiration past the effective date of the merger (the date at which the merger is expected to be completed).

Both the success probability $q$ and the fallback price $B_2$ are latent (unobserved) variables, and thus, in the absence of options traded on $B$, one cannot use only the stock price of $B$ to identify both $q$ and $B_2$. Therefore, our estimation method also uses call options traded on $B$, and thus has two—or more, if we use more than one option—observed variables to identify two latent variables.\footnote{Since the Black–Scholes formula is non-linear in the stock price, we need a statistical technique that deals with non-linear formulas and identifies both the values of the latent variables and the parameters that generate the processes. The method we use is the Markov Chain Monte Carlo (MCMC); see, e.g., Johannes and Polson (2003), or Jacquier, Johannes, and Polson (2007).}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure2}
\caption{Time Series of Median Implied Volatilities for Successful and Failed Mergers. This figure plots the time series of median Black–Scholes implied volatilities ($\sigma_{\text{implied}}$) for at-the-money call options on the target company of the 843 cash mergers announced between January 1996 and December 2014 with options traded on the target company. (Prices are the average between quoted ask and bid prices.) The time is from 20 days before to 20 days after the merger announcement date, and 20 days before the date by which the merger either succeeds or fails. We mark with “∗” the median $\sigma_{\text{implied}}$ for the deals that eventually succeeded, and with “○” the median $\sigma_{\text{implied}}$ for the deals that eventually failed.}
\end{figure}
We apply the option formula to the cash mergers announced between January 1996 and December 2014, for which there are options traded on the target company. We test our model in three different ways. First, we compare the model-implied option prices to those coming from the Black–Scholes formula, and we investigate the volatility smile. Since our estimation method uses one option each day, we check whether the prices of the other options on that day—for different strike prices—line up according to our formula. Second, we explore whether the success probabilities uncovered by our approach predict the actual deal outcomes we observe in the data. Third, we explore the implications of our model for the volatility dynamics and risk premia associated with mergers.

In comparison with the Black–Scholes formula with constant volatility, our option formula does significantly better. Indeed, the average percentage error is 36.68% for our model compared to an error of 59.69% in the case of the Black–Scholes model. In the same cross-section of firms, the third quartile of the percentage error for our model is 46.57%, compared to 71.26% for the Black–Scholes model.

Our model predicts a kink in the volatility smile, whose magnitude, as can already be observed from Figure 1, increases with the success probability. For a more formal test, instead of looking at the implied volatility plot, we consider plotting the call option price against the strike price. Then the model predicts that the magnitude of the kink normalized by the time discount coefficient should be precisely equal to the success probability. A regression of the normalized kink on the estimated success probability strongly supports the prediction that the intercept equals to 0 and the slope equals to 1. Remarkably, in our estimation procedure we use only one option each day, yet we match well the whole cross section of options for that day, and in particular, as we mentioned, we also match the magnitude of the kink.

We show that the probabilities estimated using our formula predict the outcomes of deals in the data. In particular, this method does significantly better than the “naive” method widely used in the mergers and acquisitions literature (see for instance Brown and Raymond 1986), which estimates the success probability based on the distance between the current stock price and the offer price in comparison to the pre-announcement.

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8In both cases, the error is larger than the average percentage bid-ask spread for options in our sample, which is 27.46%.
Another implication of our model is that the merger risk premium may be estimated to be proportional to the drift coefficient in the diffusion process for the success probability. This is noisy at the individual deal level, but over the whole sample the average merger risk premium is significantly positive, at an 122.05% annual rate. One explanation for this large number is that betting on merger success is a leveraged, option-like bet on the market index. Indeed, Mitchell and Pulvino (2001) find that the performance of mutual funds involved in merger arbitrage (also called “risk arbitrage”) is equivalent to writing naked put options on a market index. Nevertheless, it is possible that the merger risk premium is too high to be justified by fundamentals. If this is the case, one should expect the merger risk premium to decrease over time, as the result of more investors taking bets on mergers. In agreement with this intuition, we find that over the last five years of our sample (January 2010 to December 2014) the average annual merger risk premium has decreased to 86.47%.

Our paper is, to our knowledge, the first to study option pricing on mergers by allowing the success probability to be stochastic. This has the advantage of being realistic. Indeed, many news stories before the resolution of a merger involve the success of the merger. A practical advantage is that by estimating the whole time series of success probabilities, we can estimate for instance the merger risk premium. Also, our model is well suited to study cash mergers, which are difficult to analyze with other models of option pricing. Subramanian (2004) proposes a jump model of option prices on stock-for-stock mergers. According to his model, initially the price of each company involved in a merger follows a process associated to the success state, but may jump later at some Poisson rate to the process associated to the failure state. This approach cannot be extended to cash mergers: when the deal is successful, the stock price of the target becomes equal to the cash offer, which is essentially constant; thus,

\footnote{Dukes, Frolich and Ma (1992) consider the 761 cash mergers between 1971 and 1985 and report returns to merger arbitrage of approximately 0.47% daily, or approximately 118% annualized. See also Jindra and Walkling (2004), who obtain similar numbers but also take into account transaction costs.}

\footnote{An implication of Subramanian (2004) is that the prices of the acquirer and the target companies are perfectly correlated, which is not realistic when the merger has a low success probability. Moreover, his model implies that the success probability of a merger decreases deterministically with time, even when the merger is likely to succeed. Samuelson and Rosenthal (1986) find empirically that the success probability usually increases over time.}
the corresponding process has no volatility. In our model, the price of the target is volatile: this is due to both a stochastic success probability and a stochastic fallback price.

The literature on option pricing for companies involved in mergers is scarce, and, with the exception of Subramanian (2004), mostly on the empirical side: see Barone-Adesi, Brown, and Harlow (1994) and Samuelson and Rosenthal (1986).\textsuperscript{11} The latter paper is close in spirit to ours. They start with an empirical formula similar to our theoretical result, although they do not distinguish between risk neutral and actual probabilities. Assuming that the success probability and fallback prices are constant (at least on some time-intervals), they develop an econometric method of estimating the success probability.\textsuperscript{12} The conclusion is that market prices usually reflect well the uncertainties involved, and that the market’s predictions of the success probability improve monotonically with time.

Our paper is also related to the literature on pricing derivative securities under credit risk. The similarity with our framework lies in that the processes related to the underlying default are modeled explicitly, and their estimation is central in pricing the credit risk securities. See, e.g., Duffie and Singleton (1997), Pan and Singleton (2008), Berndt et al. (2005). Similar ideas to ours, but involving earning announcements can be found in Dubinsky and Johannes (2005), who use options to extract information regarding earnings announcements.

The paper is organized as follows. Section 2 describes the model, and derives our main pricing formulas, both for the stock prices and the option prices corresponding to the stocks involved in a cash merger. Section 3 presents the data and methodology, as well as the empirical tests of our model. Section 4 discusses the assumptions of the model and the robustness of our empirical results. Section 5 concludes. All proofs are in Appendix A.

\textsuperscript{11}Several papers show that options can be useful for extracting information about mergers, although the variable of interest in many of these paper are the merger synergies; see Hietala, Kaplan, and Robinson (2003), Barracough, Robinson, Smith, and Whaley (2013). Cao, Chen, and Griffin (2005) observe that option trading volume imbalances are informative prior to merger announcements, but not in general.

\textsuperscript{12}They estimate the fallback price by fitting a regression on a sample of failed deals between 1976–1981. The regression is of the fallback price on the offer price and on the price before the deal is announced.
2 Model

2.1 Setup

We use a continuous-time framework as in Duffie (2001, Part II). Let $W(t)$ be a 3-dimensional standard Brownian motion on a probability space $(\Omega, \mathcal{F}, P)$. The market has a risk-free security with instantaneous rate $r$, and three risky securities: $B_1$, the offer price; $B_2$, the fallback price; and $p_m$, the bet price, where the bet is placed on the success of the merger. Consider $T_e > 0$, the effective date of the merger. Then the prices of the three risky securities are Itô processes that satisfy for all $t \in [0, T_e)$,

$$
\begin{align*}
\frac{dB_i(t)}{B_i(t)} &= \mu_i(B_i(t), t)dt + \sigma_i(B_i(t), t)dW_i(t), \quad i = 1, 2, \\
p_m(t) &= e^{-r(T_e-t)}q(t), \quad \text{with} \quad q(t) = \Phi(d_{B_3,T}),
\end{align*}
$$

(1)

such that $\mu_i$ and $\sigma_i$ satisfy regularity conditions as in Duffie (2001), and $B_i > 0$ and $p_m \in (0, 1)$ almost surely. With this specification, the processes $B_1$, $B_2$ and $p_m$ are independent. The case when some of these processes are correlated is discussed in Section 4.4.

The methodology of this section works with general processes $B_1, B_2, p_m$. A particular example is given by the processes:

$$
\begin{align*}
\frac{dB_i(t)}{B_i(t)} &= \mu_i dt + \sigma_i dW_i(t), \quad i = 1, 2, 3, \\
p_m(t) &= e^{-r(T_e-t)}q(t), \quad \text{with} \quad q(t) = \Phi(d_{B_3,T})
\end{align*}
$$

(2)

where $\mu_i$ and $\sigma_i$ are constants, $\Phi(\cdot)$ is the standard normal cumulative density, and $d_{B_3,T}$ is the corresponding Black–Scholes term for pricing options on $B_3$ with expiration $T \geq T_e$ and strike $K = 1$.\footnote{When $K = 1$, equation (A5) in Appendix A implies that $d_{B_3,T} = \frac{\ln(B_3(t)) + (r - \frac{1}{2}\sigma^2_3)(T-t)}{\sigma_3\sqrt{T-t}}$.}

Note that in this example $p_m(t)$ is the price of a digital option expiring at $T$ which pays one if $B_3(T) \geq 1$ and zero otherwise.

To model cash mergers, consider a company $A$ (the acquirer) which announces at $t = 0$ that it wants to merge with a company $B$, the target. The acquisition is to be made with $B_1$ dollars in cash per share, a quantity which is not necessarily known at
At the end of the effective date $T_e$ the uncertainty about the merger is resolved, and the following quantities also become known: the offer $B_1$ and the fallback $B_2$.\footnote{In this section, the effective date is assumed fixed and known in advance by all market participants. Later, in Section 4.3 we analyze the case when $T_e$ can change before the deal is completed.} If the merger succeeds, $B_1$ is the amount that the target’s shareholders receive per share. If the merger fails, $B_2$ is the market value that the target has as an independent firm. Both values $B_1$ and $B_2$ become known at $T_e$ whether the merger is successful or not.

At each date $t$ between 0 and $T_e$, we assume that the process $p_m(t)$ in (1) is the price of a contract that pays 1 if the merger succeeds or 0 if the merger fails.\footnote{In practice, this type of contract exists in betting markets that wager on the outcome of political elections or sports games. We do not know, however, of any market that bets on the outcome of merger deals, probably because it would create opportunities for illegal insider trading.} Define the \textit{risk neutral success probability}, or simply the \textit{success probability}, to be the process

$$q(t) = p_m(t) e^{r(T_e-t)}.$$ \hspace{1cm} (3)

Note that, despite the fact that we use continuous processes, we do not require the success probability on the effective date to converge either to 0 or 1. We thus allow for the possibility of a last-minute surprise at the effective date.\footnote{One possibility is to model $q$ as a process with jumps, but to estimate the jump parameters we would need a longer time series than we typically have for merger deals. Hence, we only allow a jump at $T_e$. In practice, mergers are often decided before the effective date, and as a result in some cases the target stops trading before the effective date (in about 3% of the merger deals in our sample). In that case, in our empirical analysis we redefine the effective date as the last actual trading date. We discuss the issue of a random effective date in Section 4.3.} We define the time $T'_e$ to be the instant after $T_e$ when the uncertainty is resolved. We extend $q$ at $T'_e$ as follows: $q(T'_e) = 1$ if the merger is successful, or $q(T'_e) = 0$ if the merger fails. We also extend $B_1$ and $B_2$ at $T'_e$ by continuity: $B_i(T'_e) = B_i(T_e)$ for $i = 1, 2$.

Denote by $Q$ the equivalent martingale measure associated to $B_1, B_2$ and $p_m$, such that these processes are $Q$-martingales after discounting at the risk-free rate $r$ (see Duffie, 2001, Chapter 6).\footnote{The equivalent martingale measure is associated to fairly general processes, and it is not to be confused with the Black–Scholes risk-neutral probability. In the example in (2), the two notions happen to coincide, but in general this is not true.} Equation (3) then implies that the success probability $q$ is a $Q$-martingale. To include the final resolution of uncertainty, we extend the probability space $\Omega$ on which $Q$ is defined by including the binomial jump of $q$ at $T'_e$. This defines a new equivalent martingale measure $Q'$ and a new filtration $\mathcal{F}'$, such that $B_1$ and $B_2$,
$q$ are processes on $[0, T_e] \cup \{T'_e\}$, and $q$ is a $Q'$-martingale.

### 2.2 Option Prices and the Volatility Smile

We now compute the stock price of the target company $B$, as well as the price of a European call option traded on $B$ with strike price $K$ and expiration after the effective date, i.e., $T \geq T_e$. Recall that $B_i(t)$, $i = 1, 2$, is the market price of a security that pays $B_i(T_e)$ on the effective date, where $B_1(T_e)$ is the offer price and $B_2(T_e)$ is the fallback price. We also must specify what happens if the owner of an option receives cash before the expiration date, which is the case for instance if the merger is successful and the offer price $B_1$ is above the strike price $K$. Then, we assume that the owner invests the cash proceeds in a money market account at the risk-free $r$.

**Proposition 1.** If $B_1$, $B_2$ and $q$ are independent processes, then the target’s stock price satisfies

$$B(t) = q(t)B_1(t) + (1 - q(t))B_2(t), \quad t \in [0, T_e]. \tag{4}$$

The price of a European call option on $B$ with strike price $K$ and expiration $T \geq T_e$ is

$$C^{K,T}(t) = q(t)C^{K,T_e}_1(t) + (1 - q(t))C^{K,T}_2(t), \quad t \in [0, T_e], \tag{5}$$

where $C^{K,T}_i(t)$ is the price of the European call option with payoff $(B_i(T) - K)^+$ at $T$.

In particular, if the offer price $B_1$ is constant, Proposition 1 implies that

$$B(t) = q(t)B_1 e^{-r(T_e-t)} + (1 - q(t))B_2(t). \tag{6}$$

$$C^{K,T}(t) = q(t)(B_1 - K)^+ e^{-r(T_e-t)} + (1 - q(t))C^{K,T}_2(t) \quad \text{if } T \geq T_e. \tag{7}$$

Thus, when the sources of merger uncertainty are uncorrelated, the target stock price has a particularly simple formula. The same is true for European option prices on the target if in addition the option expires after the effective date. If instead the option expires before the effective date, the formula is more involved (see Section 4.3). When $T = T_e$ is discussed in Section 4.3.\footnote{The complementary case $T < T_e$ is discussed in Section 4.3.}
the sources of merger uncertainty are correlated, even the formula for the stock price becomes more complicated (see Section 4.4).

We now study the volatility smile, which is the Black–Scholes implied volatility curve when the stock and option prices are computed according to our model. The volatility curve plots the Black–Scholes implied volatility of the call option price against the strike price \( K \). If the Black–Scholes model were correct, the curve would be a horizontal line, indicating that the implied volatility should be a constant: the true volatility parameter. But in practice, as observed by Rubinstein (1994), the plot of implied volatility against \( K \) is convex, first going down until the strike price is approximately equal to the underlying stock price (the option is at-the-money), and then going up. This phenomenon is called the volatility “smile,” or, if the curve is always decreasing, the volatility “smirk.”

**Corollary 1.** In the context of Proposition 1, suppose the offer price \( B_1 \) is constant and that \( B_2 \) follows an exponential Brownian motion. Then, a European call option with strike \( K \) and expiration \( T \geq T_e \) exhibits a kink at \( K = B_1 \):

\[
\left( \frac{\partial C}{\partial K} \right)_{K \downarrow B_1} - \left( \frac{\partial C}{\partial K} \right)_{K \uparrow B_1} = e^{-r(T_e-t)} q(t).
\]  

(8)

The implied volatility also exhibits a kink at \( K = B_1 \):

\[
\left( \frac{\partial \sigma_{\text{impl}}}{\partial K} \right)_{K \downarrow B_1} - \left( \frac{\partial \sigma_{\text{impl}}}{\partial K} \right)_{K \uparrow B_1} = \frac{e^{-r(T_e-t)} q(t)}{\nu(B, K, r, \tau, \sigma_{\text{impl}})},
\]

(9)

where \( \nu = \frac{\partial C}{\partial \sigma} \) is the call option vega, and \( \sigma_{\text{impl}} \) is the Black–Scholes implied volatility. Moreover, for \( q(t) \) sufficiently close to 1 and \( T_e \) sufficiently close to \( T \), the slope \( \left( \frac{\partial \sigma_{\text{impl}}}{\partial K} \right)_{K \uparrow B_1} \) is negative and the slope \( \left( \frac{\partial \sigma_{\text{impl}}}{\partial K} \right)_{K \downarrow B_1} \) is positive.

Corollary 1 shows that a volatility smile arises naturally for of options on cash mergers. More precisely, the volatility curve has a kink at \( K = B_1 \) (the offer price). The magnitude of the kink (the difference between the slope of the curve on the right and left of \( K = B_1 \)) is equal to the time-discounted success probability, divided by the option vega. Moreover, the left slope is negative and the right slope is positive for most parameter values. Thus, we obtain the familiar shape for the volatility smile around \( K = B_1 \) (see also Figure 1).
Now we show that the stock price computed in (6) exhibits stochastic volatility. We define the instantaneous volatility of a positive process $B(t)$ as the number $\sigma_B(t)$ that satisfies $\frac{dB(t)}{B(t)} = \mu_B(t)dt + \sigma_B(t)dW(t)$, where $W(t)$ is a standard Brownian motion. We compute the instantaneous volatility $\sigma_B(t)$ when the company $B$ is the target of a cash merger. By using Itô calculus to differentiate equation (6) we obtain the next result.

**Corollary 2.** Suppose $B_1$ is constant, and $B_2(t)$ and $q(t)$ satisfy $\frac{dB_2(t)}{B_2} = \mu_2 dt + \sigma_2 dW_2$, and $\frac{dq}{q(1-q)} = \mu_3 dt + \sigma_3 dW_3$, where $W_2(t)$ and $W_3(t)$ are IID standard Brownian motions. Then, the instantaneous volatility of $B$ satisfies

$$\left(\sigma_B(t)\right)^2 = \left(\frac{B_1 e^{-(T_e-t)} - B_2(t)}{B(t)} q(t)(1-q(t))\sigma_1 \right)^2 + \left(\frac{B_2(t)}{B(t)} (1-q(t))\sigma_2 \right)^2. \quad (10)$$

A consequence of this corollary is that the instantaneous volatility vanishes when $q(t)$ approaches one. This is intuitive, because when the success of the merger is assured, the stock price of the target equals the cash offer, which is assumed constant for a cash merger.

Corollary 2 explains why, as we see in Figure 2, the implied volatility of the target company $B$ in a merger tends to be lower for merger deals that are eventually successful. Indeed, in these successful mergers the probability $q$ is likely to be closer to one, and therefore the implied volatility is likely to be closer to zero.

Note that Corollary 2 implies that the volatility of the target company in a merger is naturally stochastic. This is not obtained simply by assumption as in other studies, but it is a result of a model which provides economic underpinnings for stochastic volatility. By visual inspection of equation (10), we obtain an additional implication of the Corollary: deals with higher merger premia ($B_1 - B_2(0)$) tend to have higher implied volatility. This is because when the premium is higher the term $B_1 e^{-(T_e-t)} - B_2(t)$ in equation (10) is likely to be higher, which pushes up the instantaneous volatility $\sigma_B(t)$.
3 Empirical Analysis

3.1 Sample of Cash Mergers

We build a sample of all the cash mergers that were announced between January 1st, 1996 and December 31st, 2014, and have options traded on the target company. Merger data, e.g., company names, offer prices and effective dates, are from Thomson Reuters SDC Platinum. Option data are from OptionMetrics, which reports daily closing prices starting from January 1996. We use OptionMetrics also for daily closing stock prices, and for consistency we compare them with data from CRSP.

Specifically, we start by running the following session in SDC platinum: we search for all domestic mergers (deal type 1, 2, 3, 4, 11) with the following characteristics: M&A Type equal to “Disclosed Dollar Value”; target publicly traded; consideration offered in cash (category 35, 1) with Consideration Structure equal to “CASHO” (cash only); Percent of Shares Acquiror is Seeking to Own after Transaction between 80 and 100; Percent of Shares Held at Announcement less than 20 (including empty); Status either “Completed” or “Withdrawn”. This initial search produces 3298 deals. After removing deals with no option information in OptionMetrics before Announcement, there are 973 deals left. We further remove the deals with 10 options or less traded during deal (12 deals) and deal period equal to two days or less (4 deals). After reading in detail the description of the mergers in this list, we further remove the deals for which (i) the offer is not pure cash (includes the acquirer’s stock), and (ii) the target company is subject to another concurrent merger offer. There are now 843 deals left: 736 completed, and 107 withdrawn.

To create our final sample of mergers, we remove the deals for which we cannot run our estimation procedure: with deal duration of 4 days or less (2 deals); at least one day with no underlying stock prices traded on the target (1 deal); at least one day with no options quoted (14 deals); no options quoted with expiration date past the current effective date of the merger (13 deals); non-converging estimation procedure (2 deals). The final sample contains 812 deals: 711 completed, and 101 withdrawn.

19This condition is not too restrictive, since the deal announcement date must before December 31st, 2014, which should leave enough time for most mergers to be completed by October 27, 2016, the date of the SDC query.
For the final sample, we analyze in more detail the deals in which the offer price changed during the deal. The SDC field *Price Per Share* records only the last offer price. To determine the other offer prices, we examine the following fields: *Consideration* which gives a short list of the offer prices; *Synopsis* which describes this list in more detail; *History File Event* and *History File Date* which together give a list of events, including dates when the offer was sweetened (offer price went up) or amended (offer price changed, either up or down).

We also analyze the deals for which the effective date changed during the deal. The effective date of a merger is defined by SDC as the date when the merger is completed, or when the acquiring company officially stops pursuing the bid. To determine the mergers whose effective date changed during the deal, we analyze the SDC field *Tender Offer Extensions* and then read the *History File Event* and *History File Date* to extract the dates at which the tender offer got extended (even if the number of tender offers is blank, we search for “effective” or “extended”).

Table 1 reports summary statistics for our merger sample. For instance, the average deal duration (the number of trading days until the deal either succeeds or fails) is 67 days, while the maximum deal duration is 402 days. We define the *offer premium* as the percentage difference between the (cash) offer price, and the target company stock price on the day before the merger announcement. The average offer premium in our sample is 33.5%, and its standard deviation is 37.5%. The offer price changes rarely in our sample: the median number of changes is zero, and the mean is well below one (0.13). Even the 95% percentile is 1 (one change for the duration of the deal), with a maximum of 5. The effective date changes more frequently in our sample, but the median is still zero and the mean is still below one.

Table 1 includes statistics about how often options are traded on the target company. The fraction of trading days when there exists at least one option with positive trading volume is 65% for the average deal, indicating that options in our sample are illiquid.

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20 Useful information is found in the fields *Tender Offer Original Expiration Date* (which is self-explanatory) or *Tender Offer Expiration Date* (which is the last effective date on record). The *History File Event* field usually says what the new Effective Date is, but sometimes we guess it based on the following rule: if the effective date changes at a later date $T$ without any current effective date being previously announced, we consider $T$ as the earlier effective date. This makes sense, because in principle a current effective date should always be filed with the SEC.
Table 2 gives additional evidence that for the illiquidity of options. For instance, the average percentage bid-ask spread for all call options is 27.5%.

For stock prices we use closing daily prices, and for option prices we use the closing bid-ask mid-quote, i.e., the average between closing ask and bid prices. For deals that are successful, the options traded on the target company are converted into the right to receive: (i) the cash equivalent of the offer price minus the strike price, if the offered price is larger than the strike price; or (ii) zero, in the opposite case.

3.2 Methodology

For each of the cash merger deals in our sample, we record the following observed variables for the target company: (i) the effective date of the merger, $T_e$, measured as the number of trading days from the announcement; (ii) the risk-free interest rate, $r$; (iii) the cash offer price, $B_1$; (iv) the stock price of the target company, $B(t)$, on trading day $t$; (v) the price of the call options, $C_{K,T}(t)$, traded on the target company with a strike price of $K$ and expiration date $T$. When the offer price or the effective date changes, we simply change the value of $B_1$ or $T_e$ in the formula, thus essentially assuming that these changes are unanticipated.\(^{21}\)

The latent variables in this model are the success probability $q(t)$ and the fallback price $B_2(t)$, which are assumed to evolve according to:\(^{22}\)

$$\frac{dq(t)}{q(t)(1-q(t))} = \mu_1 dt + \sigma_1 dW_1(t),$$

$$\frac{dB_2(t)}{B_2(t)} = \mu_2 dt + \sigma_2 dW_2(t),$$

with independent increments $dW_1(t)$ and $dW_2(t)$, and constant coefficients $\mu_i$ and $\sigma_i$. Alternative specifications that include the case of correlated $dW_1(t)$ and $dW_2(t)$ are discussed in Section 4.

\(^{21}\)Table 1 shows that changes in $B_1$ and $T_e$ are rare. Section 4 discusses these issues in more detail.

\(^{22}\)The specification for $q$ is similar to the Black–Scholes for the stock price: $\frac{dS}{S} = \mu dt + \sigma dW(t)$, which ensures that satisfies $S > 0$. In our case we want $q \in (0, 1)$, hence the introduction of the term $(1 - q)$ in the denominator. Note that this specification is different from the example given in (2). In that example $q$ is the price of a digital option on a log-normal stock. The new specification, however, works slightly better empirically, possibly because the price of a digital option at expiration converges to either to zero or one almost surely.
With this parametrization of the success probability, the drift $\mu_1$ has a particularly useful interpretation in relation to the merger risk premium. Indeed, recall that for a price process that satisfies $dS/S = \mu(S,t) \, dt + \sigma(S,t)\, dW(t)$ the instantaneous risk premium is given by $\mathbb{E}_t(dS/S) - r \, dt = (\mu(S,t) - r) \, dt$. In the case of a merger, the merger risk premium is associated to the price $p_m(t) = q(t) \, e^{-r(T_e-t)}$ of a digital option that pays 1 if the merger is successful and 0 otherwise. The instantaneous merger risk premium is then:

$$E_t \left( \frac{dp_m}{p_m} \right) - rdt = E_t \left( \frac{dq}{q} \right) = (1-q)\mu_1 dt. \quad (13)$$

To use the formulas in Section 2, we consider at each date $t$ only the call options with expiration date $T$ larger than the current merger effective date $T_e$. Proposition 1 then implies that the stock price $B(t)$ and the call option price $C_{K,T}(t)$ satisfy, respectively, equations (6) and (7). In our empirical specification, however, we assume that these equation hold only approximately:

$$B(t) = q(t)B_1 \, e^{-r(T_e-t)} + (1-q(t))B_2(t) + \varepsilon_B(t), \quad (14)$$
$$C_{K,T}(t) = q(t)(B_1 - K)_+ \, e^{-r(T_e-t)} + (1-q(t))C_{BS}(B_2(t), K, T-t) + \varepsilon_C(t), \quad (15)$$

where $C_{BS}(S,K,T-t)$ is the Black–Scholes formula (A5) with arguments $r$ and $\sigma_2$ omitted. The errors $\varepsilon_B(t)$ and $\varepsilon_C(t)$ are IID bivariate normal:

$$\begin{bmatrix} \varepsilon_B(t) \\ \varepsilon_C(t) \end{bmatrix} \sim \mathcal{N}(0, \Sigma_\varepsilon), \quad \text{where} \quad \Sigma_\varepsilon = \begin{bmatrix} \sigma_{\varepsilon,B}^2 & 0 \\ 0 & \sigma_{\varepsilon,C}^2 \end{bmatrix}. \quad (16)$$

In the main empirical specification, on each day $t$ we select only the option with maximum trading volume on that day.\textsuperscript{24}

Equations (11), (12), (14), (15) and (16) define a state space model with observables

\textsuperscript{23}Equation (13) implies that the merger risk premium is near zero when $q$ is near one. This comes from the functional specification of $q$ in (11), which implies that $dq/q$ has almost zero volatility (and hence it is almost risk free) when $q$ approaches one.

\textsuperscript{24}If all option trading volumes are zero on that day, use the option with the strike $K$ closest to the strike price for the most currently traded option with maximum volume. Finally, if none of these criteria are met, we simply choose the at-the-money option on that day.
$B(t)$ and $C(t)$, latent (state) variables $q(t)$ and $B_2(t)$, and model parameters $\mu_1$, $\sigma_1$, $\mu_2$, $\sigma_2$, $\sigma_{\varepsilon,B}$ and $\sigma_{\varepsilon,C}$. We adopt a Bayesian approach and conduct inference by sampling from the joint posterior density of state variables and model parameters given the observables.

Specifically, we use a Markov Chain Monte Carlo (MCMC) method based on a state space representation of our model. In this framework, the state equations (11) and (12) specify the dynamics of latent variables, while the pricing equations (14) and (15) specify the relationship between the latent variables and the observables. The addition of errors with distribution described by (16) in the pricing equations is standard practice in state space modeling; this also allows us to easily extend the estimation procedure to multiple options and missing data. This approach is one of several (Bayesian or frequentist) suitable for this problem and is not new to our paper. For a discussion, see, e.g., Johannes and Polson (2003) or Koop (2003). The resulting estimation procedure is described in detail in Appendix B.\textsuperscript{25} All priors used in our estimation are flat.

### 3.3 Empirical Results

As described in the data section, our sample contains 812 cash mergers during 1996–2014. Table 3 reports information on the ten largest merger deals sorted on the offer value, five that succeeded and five that failed.\textsuperscript{26} Figure 4 reports the estimated success probability $q$ for these ten deals. We see that large $q$ corresponds to merger success (the five deals in the left column), while small $q$ corresponds to merger failure (the five deals in the right column).

Before we discuss in more detail the success probability estimates, we analyze how well our pricing formulas (14) and (15) work. Recall that the pricing formulas for the stock price $B(t)$ and option price $C(t)$ hold with errors $\varepsilon_B(t)$ and $\varepsilon_C(t)$, respectively. The fitted values are our estimates for the stock price $\hat{B}(t)$ and the option price $\hat{C}(t)$. Table 4 reports summary statistics for the average stock pricing error $\frac{1}{T_e} \sum_{t=1}^{T_e} \left| \frac{\hat{B}(t) - B(t)}{B(t)} \right|$ over the duration of the deal. The stock pricing errors are in general very small, with a median error of 3.2 basis points.

\textsuperscript{25}As noted by Johannes and Polson (2003), equations of the type (14) or (15) are a non-linear filter. The problem is that it is difficult to do the estimation using the actual filter. Instead MCMC is a much cleaner estimation technique, but it does smoothing, because it uses all the data at once.

\textsuperscript{26}The offer value is defined as the offer price multiplied by the target’s number of shares outstanding.
We illustrate graphically the performance of the option pricing model, by selecting the deal with the largest offer premium (75.44%) among the ten mergers in Table 3. The target company of this deal is AT&T Wireless, with ticker AWE. Figure 5 illustrates for the company AWE how our model fits the call option prices, including the kink in the implied volatility curve. This is remarkable, as our estimation method only uses one option per day, yet the model is capable of accurately predicting the whole cross section of call option prices for each day.

Table 5 compares option pricing errors from two models. The first model is the one described in this paper, denoted “BMR” for short. The second model, denoted by “BS,” is the Black and Scholes (1973), with the volatility parameter \( \sigma = \bar{\sigma}_{ATM} \) given by the average implied volatility for the at-the-money (ATM) call options over the duration of the deal.

Panel A reports absolute errors (in U.S. dollars), while Panel B reports the relative errors (in percentages). Observed prices are computed as the bid-ask mid-quote. Each type of error is computed by restricting the sample of call options based on the moneyness of the option, i.e., the ratio of the strike price \( K \) to the underlying stock price \( B(t) \). We consider the following moneyness categories: (i) all call options; (ii) deep-in-the-money (Deep-ITM) calls, with \( K/B < 0.9 \); (iii) in-the-money (ITM) calls, with \( K/B \in [0.9,0.95) \); (iv) near-in-the-money (Near-ITM) calls, with \( K/B \in [0.95,1] \); (v) near-out-the-money (Near-OTM) calls, with \( K/B \in [1,1.05) \); (vi) out-of-the-money (OTM) calls, with \( K/B \in [1.05,1.1] \); and (vii) deep-out-of-the-money (Deep-OTM) calls if \( K/B > 1.1 \).

As we are interested in comparing the volatility smile for the two models, we ensure that each day the moneyness categories have sufficiently many options. Thus, suppose there are \( N_{m,t} \) options in the moneyness category \( m \) that are quoted on day \( t \). Then, to include these options in our calculations, we require \( N_{m,t} \geq 6 \) when \( m \) is the first category (all calls), and \( N_{m,t} \geq 2 \) for all the other categories (calls of a particular moneyness). Otherwise, the data corresponding to these options on day \( t \) is considered

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27 The offer premium is defined as the ratio \( \frac{B_1 - B(-1)}{B(-1)} \), where \( B_1 \) is the offer price and \( B(-1) \) is the target's stock price one trading day before the merger announcement date.

28 The moneyness intervals are chosen following Bakshi, Cao, and Chen (1997), except that we use a larger step (0.05) than their step (0.03). The reason is that they study S&P 500 index options, which are much more liquid than the options on the individual stocks in our sample.
as missing.\textsuperscript{29}

To understand how the relative pricing errors are computed in the table, consider for instance the results corresponding to the sixth moneyness category in Panel B (OTM calls). By looking at the number of observations, we see that there are only 54 target stocks for which there is at least one day when two or more OTM calls are quoted. Then, for a target stock \( j \) and for a call option on \( j \) with (bid-ask mid-quote) price \( C(t) \) quoted on day \( t \) on a stock with (closing) price \( B(t) \) and with strike \( K \), we compute the percentage pricing error by

\[
\left| \frac{C_M(t) - C(t)}{C(t)} \right|
\]

where \( C_M(t) \) is the model-implied option price, where the model \( M \) can be either the theoretical model in this paper, denoted by “BMR,” or the Black–Scholes model, denoted by “BS” for which the volatility parameter is the average at-the-money implied volatility. We then compute the average error \( e_j \) over this particular category of options on stock \( j \). The table then reports various statistics of \( e_j \) over the cross section of 54 target stocks with non-missing OTM option data.

Overall, our model does much better than the Black–Scholes model, for the whole sample, as well as for the various subsamples of options. For instance, in Panel A we see that, for all call options the average pricing error is 0.13 for the BMR model, and 0.27 for the BS model.\textsuperscript{30} Panel B shows the corresponding values for the relative pricing errors: 34.03% for the BMR model, and 60.03% for the BS model.

In Table 6 we consider pricing errors of our model by considering the call price instead of its implied volatility. The table presents summary statistics for the ratio of the absolute pricing error of a call option to its bid-ask spread. We are particularly interested in ratios larger than one: if our model produces the arbitrage-free correct price for the call option, then a pricing error ratio larger than one indicates a potential arbitrage opportunity. We see that arbitrage opportunities are rare but they do exist: the 95% percentile for the pricing error ratio produces numbers larger than one for several moneyness categories (ITM, NITM, NOTM, OTM, DOTM).

We test the implications of the model regarding the kink in the volatility smile,\textsuperscript{29}As we see in Table 5, imposing \( N \geq 2 \), e.g., for ITM calls reduces to 59 (out of 812) the number of firms for which there is at least one day \( t \) with two or more quoted ITM calls. The higher daily threshold \( N \geq 6 \) for all calls is chosen so that there are enough firms (470) satisfying this restriction. Choosing a different threshold produces similar results.

\textsuperscript{30}Comparing these results to those of Table 2, it appears that the average pricing error of our model is well within the average bid-ask spread, which is 1.08 for all calls.
which can be observed in a particular case in Figure 5. Corollary 1 shows that this kink corresponds to a kink in the plot of the call option price against the strike price. Moreover, it shows that this kink, normalized by the time discount coefficient, should be equal to the success probability \( q \). Empirically, if we do an OLS regression of the normalized kink on the estimated success probability, we should find that the slope coefficient is equal to one. Table 7 displays results of OLS regressions of the estimated kink (the difference in slopes above and below the strike price closest to the offer price) and the estimated success probability using our methodology which uses only one option each day. We also regress a modified kink \( \tilde{C}^{\text{kink}} \) on a modified success probability \( \tilde{q}(t) \), where the modification involves truncating the values of the kink to be in \((0, 1)\) and inverting them via the standard normal cumulative density function.\(^{31}\) Overall, we find that the regressions coefficients are indeed close to one.

In addition to pricing options, we also check whether the estimates of state variables recovered using our model are economically meaningful. We begin by asking whether the success probabilities estimated by our model predict the outcomes of deals in the sample. Figure 4 illustrates the results for the ten largest deals from Table 3, five of which succeeded, and five of which failed. Figure 4 displays the time series of the posterior median together with a 90% credibility interval (i.e., the 5\(^{\text{th}}\), 50\(^{\text{th}}\), and 95\(^{\text{th}}\) percentiles of the posterior) for the time series of the state variable \( q(t) \). The estimates of \( q(t) \) for the five deals that succeeded—on the left column—are overall much higher than for the five deals that failed—on the right column.\(^{32}\)

Table 8 shows that in general our estimates of \( q \) predict well the outcome of the deal. We choose 10 evenly spaced days during the period of the merger deal: for \( n = 1, \ldots, 10 \), choose \( t_n \) as the closest integer strictly smaller than \( n \frac{T}{10} \). The Table reports the pseudo-\( R^2 \) for 10 cross-sectional probit regressions of the deal outcome (1 if successful, 0 if it failed) on \( q(t_n) \). \( R^2 \) increases approximately from 10% to about 47%, which indicates that, the closer one comes to the effective date, the better the success probability predicts the outcome of the merger. Note that we do not impose the success probability to be 0

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\(^{31}\)We perform this modification so that their distribution is closer to being normal.

\(^{32}\)This is the only place we use all the available options to estimate the state variables. If instead we select one option each day (the call option with the maximum trading volume on that day), then the results still hold but the error bars are wider, and the contrast between the two groups is not as strong.
or 1 at the effective date $T_e$. This would likely lead to an even better fit.

We contrast our model-implied success probability to the “naive” method of Brown and Raymond (1986), which is used widely in the merger literature. This is defined by considering the current price $B(t)$ of the target company. If this is close to the offer price $B_1$, the naive probability is high. If instead $B(t)$ is close to the pre-announcement stock price $B_0(t)$, then the naive probability is low. Specifically define $q_{\text{naive}}(t) = \frac{B(t) - B_0}{B_1 - B_0}$ if $B_0 < B(t) < B_1$. If $B(t) < B_0$ (or $> B_1$), $q_{\text{naive}}$ is set equal to zero (one). Table 8 reports the results from a panel probit regression of the deal outcome on estimated success probability, either during the whole time period of the deal or only during the second half. We see that our estimation method produces a larger pseudo-$R^2$ in both cases than the naive method.

Our model has implications for measurement of the volatility of the target firms. Corollary 2 shows that the return volatility of the target company $B$ approaches zero when the deal is close to completion, i.e., the success probability $q$ is close to one. This explains the empirical fact when a merger is close to completion, the Black–Scholes implied volatility of the target company converges to zero. This fact can be seen in the data in Figure 2.

Finally, we explore the possibility to estimate the merger risk premium using the drift coefficient in the diffusion process for the success probability (11). According to equation (13), the instantaneous merger risk premium equals $(1 - q)\mu_1 dt$. In practice, we take the merger risk premium over by averaging out $1 - q$ over the life of the deal: $(\bar{1} - \bar{q})\mu_1$. The individual estimates for $(\bar{1} - \bar{q})\mu_1$ are very noisy, but over the whole sample the average merger risk premium is significantly positive, and the annual figure is 122.05%. This number is very large but is consistent with the existing literature.\footnote{See, e.g., Dukes, Frolich and Ma (1992), who report an average daily premium of 0.47% (approximately 118% annualized), over 761 cash mergers between 1971 and 1985. See also Jindra and Walkling (2004), who confirm the results for cash mergers, but also take into account transaction costs; and Mitchell and Pulvino (2001), who consider the problem over a longer period of time, and for all types of mergers.} One potential explanation for the large estimate is that merger arbitrage (which is also called “risk arbitrage”) is a leveraged, option-like bet on the market index (this intuition is consistent with Mitchell and Pulvino 2001). Nevertheless, it is possible that the merger risk premium is too high to be justified by fundamentals. If this is the case,
some investors who are aware of an excessive risk premium should have entered the
merger arbitrage business, and as a result they should have brought down the premium.
To check whether this is the case, we restrict our merger sample to the last five years,
when the announcement date is between January 2010 and December 2014. For this
subsample we see that indeed the estimated premium is significantly lower: 86.47%.

4 Discussion and Robustness

After considering various alternative specifications of our baseline model, our model
appears to be essentially robust. One explanation for this robustness is that only a
simple model can address the large amount of noise present in option prices on individual
companies. Table 2 shows that the average percentage bid-ask spread of call options
written on the target companies in our cash merger sample is 27.46%. Thus, attempts
to impose any additional structure on our baseline model typically result in increasing
the noise in our estimates, but do not significantly change our main results.

4.1 Assumptions on Observed Variables

In our baseline model, we assume that the effective date $T_e$ is known from the beginning.
As a consequence, in our empirical tests we have taken $T_e$ to be the date when the
merger is either successful or fails. But, in reality it is often the case that the effective
date of the merger subsequently changes from the initial date reported on the merger
announcement day. (See Table 1 for some statistics regarding the number of effective
date changes.) We address this concern in several ways. First, in contrast with our
baseline specification in which we consider options with shortest expiration date after
$T_e$, we change our specification to include call options with longer maturity. When we
do this, the results stay essentially the same. Second, in Section 4.3 we show that by
considering options with expiration date before $T_e$, the errors are not very large.

In our empirical methodology, we also assume that when the cash offer price $B_1$
changes, it gets simply replaced in the formulas with the new value. That is, if we
denote by $\tilde{B}_1(t)$ the time series of the offer price, we use equation (6) to write the stock
price as $B(t) = q(t)\tilde{B}_1(t) e^{-r(T_e-t)} + (1 - q(t))B_2(t)$. Alternatively, if that the market
knows that $B_1$ is stochastic (but independent from the other processes), Proposition 1 implies that $B(t) = q(t)B_1(t) + (1 - q(t))B_2(t)$, where $B_1(t)$ is the market price at $t$ of a contingent security that pays the (random) offer price on the effective date $T_e$. Thus, as long as $B_1(t)$ stays close to the discounted value of the current offer price $(e^{-r(T_e-t)} \tilde{B}_1(t))$, the model error is small. In the end, the validity of our approach depends on the perceived volatility of $\tilde{B}_1$ being close to zero. Table 1 shows that this is a plausible assumption, since the number of offer price changes is usually zero.\footnote{In principle, we can modify our model in order to be able to estimate the volatility of $B_1$. We have not pursued this avenue, however, since $B_1$ does not change often enough to allow proper statistical identification.}

Other observed variables are the target stock price $B(t)$, and the call option price $C(t)$. In an alternative specification, instead of the usual bid-ask mid-quote, we use the ask price. This does not change our results, except that the option pricing errors in Table 5 corresponding to our model become somewhat larger, although still significantly smaller than the errors corresponding to the Black–Scholes model.

### 4.2 Alternative Specifications for the Latent Variables

Alternative specifications for the success probability $q(t)$ show that our model is robust. If $X_1(t)$ is the Itô process $dX_1(t) = \mu_1 dt + \sigma_1 dW_1(t)$ with constant coefficients $\mu_1$ and $\sigma_1$, we consider the following functional forms for the success probability: (i) $q(t) = \frac{e^{X_1(t)}}{1 + e^{X_1(t)}}$, (ii) $q(t) = E_t^Q(1_{X_1(T)>0}) = \Phi\left(\frac{X_1(t) + (r - \frac{1}{2}\sigma_1^2)(T-t)}{\sigma_1 \sqrt{T-t}}\right)$, which is the price of a digital option on $X_1$, i.e., the bet that $X_1(T) > 0$. We further consider the specification (i) but when $X_1(t)$ a jump process of the form $dX_1(t) = \mu_1 dt + \sigma_1 dW_1(t) + ZdJ$, where $Z$ is the jump size and $J$ is a Poisson process. Furthermore, our estimation method allows direct constraints on $X_1(t)$. For instance, if we require $X_1(t) \in [-1, 1]$, we consider the following functional forms: (iii) $q(t) = \frac{1}{2}(1 + \frac{X_1}{(1-a)+a|X_1|})$, with $a \in (0, 1)$, (iv) $q(t) = \frac{1}{2} + X_1 - (\frac{a}{4} + \frac{1}{2})X_1|X_1| + \frac{a}{4}X_1^3$, with $a \in (0, 1)$.\footnote{E.g., the function $f(x) = \frac{1}{2}(1 + \frac{x}{(1-a)+a|x|})$ satisfies $f(-1) = 0$, $f(0) = \frac{1}{2}$, $f(1) = 1$, $f'(-1) = f'(1) = 1 - a \frac{1}{2}$. Thus, when $a \approx 1$, $f'(-1) = f'(1) \approx 0$, which has the practical consequence that, once $q(t)$ is estimated to be close to 0 or 1, it is more likely to stay near those values.} Using these alternative specifications, our results do not change significantly, although in some cases such as (i) the algorithm converges much less often, and thus no estimates are obtained.\footnote{This is because $q(t) = \frac{e^{X_1(t)}}{1 + e^{X_1(t)}}$ is nearly constant for $X_1$ large. Thus, when $X_1$ by chance drifts towards large values, the estimated posterior density is essentially flat and cannot distinguish between
One important issue is that the success probability and the fallback price might in reality be correlated. To address this issue, we consider the usual functional specifications for $q(t)$ and $B_2(t)$, except that the corresponding Itô increments $dW_1$ and $dW_2$ are no longer independent, but instead have a bivariate normal distribution, with instantaneous correlation $\rho \in [-1, 1]$. Then, as we see in Section 4.4, the formulas for the stock and option prices become more complicated and involve numerical integration. This slows down the algorithm considerably, by a factor of at least 100. Nevertheless, when we perform the estimation for the 10 companies in Table 3, we note that in all cases the parameter $\rho$ is poorly identified, i.e., its estimated posterior likelihood is essentially flat. We interpret this result as indicating that a priori it is not even clear what sign the correlation $\rho$ should have. For instance, under normal circumstances, small changes in $B_2$ are not likely to affect $q$, while larger changes in $B_2$ in either direction may actually decrease $q$, thus pointing to a potentially non-linear relationship between $q$ and $B_2$. We are skeptical of pursuing such extensions, since they are likely to lead to poor statistical identification.

Finally, because stock price errors are relatively small (see Table 4), we may assume that equation (14) holds without error, i.e., $\varepsilon_B(t) = 0$. In that case the success probability $q(t)$ can be expressed as a function of the fallback price $B_2(t)$ and substituted in (15). To simplify formulas, denote by $B_1(t) = B_1 e^{-r(T_e-t)}$, $C_1(t) = (B_1 - K)_+ e^{-r(T_e-t)}$, and $C_2(t) = C_{BS}(B_2(t), K, r, T - t, \sigma_2)$, where $C_{BS}$ is the Black–Scholes formula (A5). Then equations (14) and (15) imply

$$q(t) = \frac{B(t) - B_2(t)}{B_1(t) - B_2(t)}, \quad C(t) = C_2(t) + \frac{B(t) - B_2(t)}{B_1(t) - B_2(t)} (C_1(t) - C_2(t)) + \eta_C(t). \quad (17)$$

By focusing only on the second equation, we avoid choosing a functional specification for $q(t)$, such as (11). But we must satisfy the constraint $q(t) \in [0, 1]$, which means that $B_2(t)$ must be selected so that $\frac{B(t) - B_2(t)}{B_1(t) - B_2(t)} \in [0, 1]$. The results that use this specification are much noisier than those obtained under our baseline specification, while they do not significantly change our results.
4.3 Options Expiring Before the Effective Date

In this section we use the same setup as in Section 2.1, except that we consider options that expire before the effective date. To simplify the presentation, we assume that (i) $B_1$ is constant, (ii) $B_2(t)$ is a log-normal process with constant coefficients, and (iii) $p_m(t)$ is the price of a digital option that pays one if $B_3(T_e)$ is above $K_3$ and zero otherwise, where $B_3(t)$ is a log-normal process with constant coefficients. Specifically, $B_2(t)$ and $B_3(t)$ satisfy

$$d B_i(t) = \mu_i B_i(t) dt + \sigma_i B_i(t) dW_i(t), \quad i = 2, 3. \tag{18}$$

Using the risk neutral Black–Scholes formalism we write for any $T \geq t$

$$B_i(T) = B_i(t) \exp \left( (r - \frac{\sigma_i^2}{2})(T - t) + \sigma_i \sqrt{T - t} \varepsilon_i \right), \quad i = 2, 3, \tag{19}$$

where $\varepsilon_2$ and $\varepsilon_3$ have independent standard normal distributions. Also, the risk neutral probability $q$ satisfies for any $t < T < T_e$

$$q(T) = \Phi \left( \frac{\sqrt{T - t} \Phi^{-1}(q(t)) + \sqrt{T - t} \varepsilon_3}{\sqrt{T_e - T}} \right), \tag{20}$$

where $\varepsilon_3$ has a standard normal distribution.³⁷

The next result computes the price of European calls that expire before $T_e$.

**Proposition 2.** Suppose $B_1$ is constant, and $B_2$ and $q$ are independent processes. Define

$$B_1(T) = B_1 e^{-(T_e - T)}, \quad \varepsilon = \begin{cases} \frac{\sqrt{T - t}}{\sqrt{T - t}} \Phi^{-1} \left( \frac{K}{B_1(T)} \right) - \frac{\sqrt{T - t}}{\sqrt{T - t}} \Phi^{-1}(q(t)), & \text{if } K < B_1(T), \\ +\infty, & \text{if } K \geq B_1(T) \end{cases} \tag{21}$$

³⁷See the proof of Proposition 2.
Then the price of a European call option on \( B \) with strike \( K \) and expiration \( T < T_e \) is

\[
C(t) = \int_{-\infty}^{\varepsilon} \left[ (q(T)B_1(T) - K) e^{-r(T-t)} + (1 - q(T)) B_2(t) \right] \phi(\varepsilon_3) \, d\varepsilon_3 \\
+ \int_{\varepsilon}^{+\infty} \left[ (q(T)B_1(T) - K) e^{-r(T-t)} \Phi\left(d_-(B_2(t), \frac{K-q(T)B_1(T)}{1-q(T)}, T-t)\right) \\
+ (1 - q(T)) B_2(t) \Phi\left(d_+(B_2(t), \frac{K-q(T)B_1(T)}{1-q(T)}, T-t)\right) \right] \phi(\varepsilon_3) \, d\varepsilon_3.
\]

where \( q(T) \) is the function of \( \varepsilon_3 \) described in (20), and \( d_{\pm}(S, K, T-t) \) are as in the Black–Scholes formula (A5) with arguments \( r \) and \( \sigma^2 \) omitted.

**Figure 3: Pricing Calls with Expiration before the Effective Date.** This figure displays pricing errors for European call options on the target company \( B \) of a merger, when the expiration date \( T \) is smaller than the merger effective date \( T_e \). The figure compares prices compute at \( t = 0 \): (i) the correct price \( C_{\text{correct}} \) from equation (22), and (ii) the “simple” price \( C_{\text{simple}} \) from equation (7) which is correct only if \( T \) is larger than \( T_e \). On the \( x \)-axis is the time \( T \) until expiration, and on the \( y \)-axis is the pricing error \( (C_{\text{simple}} - C_{\text{correct}})/C_{\text{correct}} \). The parameters used in the formulas are: \( T_e = 100 \) days (effective date) divided by 252 = the number of trading days in 1 year; \( B_1 = 100 \) (offer price); \( B_2(0) = 90 \) (fallback price at \( t = 0 \)); \( q(0) = 0.5 \) (success probability at \( t = 0 \)); \( r = 0.05 \) (annual risk-free rate); \( \sigma^2 = 0.4 \) (annual fallback price volatility). The stock price at \( t = 0 \) corresponding to these parameters is \( B(0) = 94.018 \). The figure display the results when the expiration date \( T \) varies between 80 and 100 days, and the strike price \( K \) varies between 75 and 105 (\( K = 95 \) corresponds to the at-the-money call).
In Figure 3 we compare the correct price in (22) to the price from equation (7), which is correct only if \( T > T_e \). The purpose of this exercise is to see whether a stochastic effective date significantly affects the option price. Suppose at \( t = 0 \) the current effective date of the merger is in \( T_e = 80 \) days, and we consider a European call option with strike \( K = 95 \) and expiration in \( T = 90 \) days.\(^{38}\) Then, formula (7) implies that the “simple” call price is \( C_{\text{simple}} = 6.0408 \). If the effective date suddenly changes to \( T_e = 100 \), the option expires before \( T_e \), and equation (22) implies that \( C_{\text{correct}} = 5.4906 \). If instead we use the formula (7), which is no longer correct, we have \( C_{\text{simple}} = 6.0360 \), which is slightly smaller than before (by 0.08%) because the time value of the cash offer, which is now expected later. The corresponding pricing error is \( (C_{\text{simple}} - C_{\text{correct}})/C_{\text{correct}} = 9.9336\% \), which in Figure 3 corresponds to the point on the curve \( K = 95 \) with \( x \)-coordinate \( T = 90 \). This error, however, assumes that the effective date changes with certainty at \( t = 0 \). If instead the probability of an effective date change is smaller, e.g., 1% per day, then the actual pricing error is smaller than 9.9336\%.\(^{39}\)

### 4.4 Correlated Latent Variables

In this section we use the same setup as in Section 2.1, except that the success probability and the fallback price are correlated. To simplify the presentation, we assume that (i) \( B_1 \) is constant, (ii) \( B_2(t) \) is a log-normal process with constant coefficients, (iii) \( p_m(t) \) is the price of a digital option that pays one if \( B_3(T_e) \) is above \( K_3 \) and zero otherwise, where \( B_3(t) \) is a log-normal process with constant coefficients, and (iv) the contemporaneous increments of \( \ln(B_2(t)) \) and \( \ln(B_3(t)) \) have a bivariate normal distribution with instantaneous correlation \( \rho \in (-1, 1) \). Specifically, \( B_2(t) \) and \( B_3(t) \) are defined as in (18), and therefore satisfy the same formulas as in (19). The difference is that now \( \varepsilon_2 \) and \( \varepsilon_3 \) are no longer independent but have a bivariate normal distribution with density

\[
 f_{\rho}(x, y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{x^2 + y^2 - 2\rho xy}{2(1-\rho^2)}\right).
\]

\(^{38}\)The parameters values are: \( B_1 = 100, B_2(0) = 90, q(0) = 0.5, r = 0.05 \). The number of days is annualized by division with 252, which by convention is the number of trading days in one year.

\(^{39}\)Table 1 shows that the effective date changes on average 0.74 times for a deal duration which on average is 66.6 trading days, hence the probability of an effective date change during any particular day is 0.74/66.6 ≈ 1.11\%. 

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The next result extends the formulas (6) and (7) to the new context.

**Proposition 3.** If $B_1$ is constant, and $B_2$ and $q$ are correlated processes as above, then the target’s stock price satisfies for $t < T_e$

$$B(t) = q(t)B_1 e^{-r(T_e - t)} + \left(1 - \Phi\left(\Phi^{-1}(q(t)) + \rho\sigma_2\sqrt{T - t}\right)\right)B_2(t). \quad (24)$$

The price of a European call option on $B$ with strike price $K$ and expiration $T \geq T_e$ is

$$C^{K,T}(t) = q(t)(B_1 - K)_+ e^{-r(T_e - t)} + C_{BS}(B_2(t), T - t) \quad (25)$$

$$- \int_{-\infty}^{+\infty} \Phi\left(\frac{\Phi^{-1}(q(t)) + \rho\varepsilon_2}{\sqrt{1 - \rho^2}}\right) C_{BS}\left(B_2(t) \exp\left((r - \frac{\sigma^2}{2})(T_e - t) + \sigma_2\sqrt{T_e - t} \varepsilon_2, T - T_e\right) \phi(\varepsilon_2) \, d\varepsilon_2,$$

where $C_{BS}(S, T - t)$ is the Black–Scholes formula (A5) with arguments $K$, $r$ and $\sigma_2$ omitted.

## 5 Conclusion

In an empirical study of cash mergers since 1996, we find that equity options on firms that are the target of a cash merger display a pronounced pattern in their implied volatility smile. We call this pattern the *merger volatility smile*, and we find that it is more pronounced when the merger is close to being successful. To address this empirical regularity, we propose an arbitrage-free option pricing formula on companies that are subject to merger attempts.

Theoretically, our formula predicts a *kink* in the implied volatility curve at the cash offer price, or equivalently a kink in the call option price. Furthermore, the magnitude of the price kink equals essentially equals the (risk neutral) success probability. Empirically, we find essentially a one-to-one relationship between the magnitude of the price kink and the merger’s success probability, which confirms our theoretical prediction.

Our option formula matches option prices significantly better than the standard Black–Scholes formula. In addition, we use the resulting formula to estimate several variables of interest in a cash merger, i.e., the success probability and the fallback price. The estimated success probability turns out to be a good predictor of the deal outcome,
and it does better than the naive method which identifies the success probability solely based on how the current target stock price is situated between the offer price and the pre-merger announcement price. Besides the success probability itself, we also estimate its drift parameter, which turns out to be related to the merger risk premium. The average merger risk premium in our sample is about 122% annually, which is consistent with the cash mergers literature, although there is evidence the premium has decreased substantially in the past five years.

Our methodology is flexible enough to incorporate other existing information, such as prior beliefs about the variables and the parameters of the model. It can also be used to compute option pricing for “stock-for-stock” mergers or “mixed-stock-and-cash” mergers, where the offer is made using the acquirer’s stock, or a combination of stock and cash. In that case, it can help estimate the synergies of the deal. The method can in principle be applied to other binary events, such as bankruptcy or earnings announcements (matching or missing analyst expectations), and is flexible enough to incorporate other existing information, such as prior beliefs about the variables and the parameters of the model.

Appendix

Appendix A. Proofs of Results

Proof of Proposition 1. Recall that \( T'_e \) is the instant after \( T_e \) when the merger uncertainty is resolved, and \( Q' \) is the extension of the equivalent martingale measure \( Q \) to \([0, T_e] \cup \{ T'_e \} \). As \( q(T'_e) \) is either 1 or 0 depending on whether the merger is successful or not, the target’s payoff at \( T'_e \) is

\[
B(T_e) = q(T'_e)B_1(T_e) + (1 - q(T'_e))B_2(T_e), \quad (A1)
\]

We apply Theorem 6J in Duffie (2001) for redundant securities. Markets are dynamically complete before \( T_e \), because the uncertainty stems from the three Brownian motions involved in the definition of the securities \( B_1, B_2, \) and \( p_m \). Moreover, at \( T_e \), the stock
price has a binary uncertainty that can be spanned only by the bond and $p_m$. Then, in the absence of arbitrage, any other security whose payoff depends on $B_1$, $B_2$, $p_m$ is a discounted $Q'$-martingale. In particular, the price of the target company $B(t)$ is a discounted $Q'$-martingale, that is,

\[
B(t) = e^{-r(T_e-t)} \mathbb{E}^Q_t \left[q(T_e')B_1(T_e') + (1-q(T_e'))B_2(T_e')\right] = \mathbb{E}^Q_t (q(T_e')) e^{-r(T_e-t)} \mathbb{E}^Q_t (B_1(T_e')) + \mathbb{E}^Q_t (1-q(T_e')) e^{-r(T_e-t)} \mathbb{E}^Q_t (B_2(T_e')) \tag{A2}
\]

where for the second equation we use the independence of $B_1$, $B_2$ and $q$. This proves (4).

Consider a European call option on $B$ with strike price $K$ and maturity $T \geq T_e$. Denote by $C^{K,T}(t)$ its price at $t \leq T_e$, and by $X = \max\{X,0\}$. Denote by $C_i^{K,T}(t)$ the price of a European call option on $B_i$ with strike price $K$ and maturity $T$. Consider the payoff of the call at $t = T \geq T_e$:

\[
q(T_e')(B_1(T_e') - K)_+ e^{T-T_e} + (1-q(T_e'))(B_2(T) - K)_+. \tag{A3}
\]

where by assumption all cash obtained after the effective date $T_e$ is invested at the risk-free rate $r$. By a similar calculation as above, the call price at $t$ is

\[
C^{K,T}(t) = e^{-r(T-t)} \mathbb{E}^Q_t \left[q(T_e')(B_1(T_e') - K)_+ e^{r(T-T_e)} + (1-q(T_e'))(B_2(T) - K)_+\right] = q(t)C_i^{K,T}(t) + (1-q(t))C_2^{K,T}(t), \tag{A4}
\]

which proves (5).

**Proof of Corollary 1.** From (7) the call price is $C^{K,T}(t) = q(t) e^{-r(T_e-t)}(B_1 - K)_+ + (1-q(t))C_2^{K,T}(t)$. Since $B_2$ follows a log-normal process, the option price $C_2^{K,T}(t)$ satisfies the Black–Scholes equation and is thus differentiable in $K$ (see more details below). We now differentiate $C^{K,T}(t)$ with respect to $K$ to the left and to the right of $K = B_1$: \[
\left(\frac{\partial C}{\partial K}\right)_{K \uparrow B_1} = -q(t) e^{-r(T_e-t)} - (1-q(t)) \frac{\partial C_2^{K,T}(t)}{\partial K} \quad \text{and} \quad \left(\frac{\partial C}{\partial K}\right)_{K \downarrow B_1} = -(1-q(t)) \frac{\partial C_2^{K,T}(t)}{\partial K}.
\]
The kink is the difference $\left(\frac{\partial C}{\partial K}\right)_{K \downarrow B_1} - \left(\frac{\partial C}{\partial K}\right)_{K \uparrow B_1} = q(t) e^{-r(T_e-t)}$. This proves (8) if we can show that $C_2^{K,T}(t)$ is differentiable in $K$. To see this, consider the particular case
when \( B_2(t) \) is an exponential Brownian motion with drift \( \mu_2 \) and volatility \( \sigma_2 \). Then according to the Black–Scholes formula, we have

\[
C^K,T_2(t) = C_{BS}(B_2(t), K, r, T - t, \sigma_2) = B_2(t) \Phi(d_+) - K e^{-r(T-t)} \Phi(d_-),
\]

\[
d_\pm = \frac{\ln(B_2(t)/K) + (r \pm \frac{1}{2} \sigma_2^2)(T-t)}{\sigma_2 \sqrt{T-t}}.
\]  

(A5)

One now verifies that \( \frac{\partial C^K,T_2(t)}{\partial K} = -e^{-r(T-t)} \Phi(d_-) \), which is continuous in \( K \).

Let \( \tau_e = T_e - t \) and \( \tau = T - t \). By definition, the Black–Scholes implied volatility \( \sigma_{impl} \) of \( C^K,T_2(t) \) satisfies \( C^K,T_2(t) = C_{BS}(B(t), K, r, \tau, \sigma_{impl}) \), where \( C_{BS} \) is the Black–Scholes formula (A5). By differentiating \( C(t) = C_{BS}(B(t), K, r, \tau, \sigma_{impl}) \) with respect to \( K \), we get

\[
\left( \frac{\partial \sigma_{impl}}{\partial K} \right)_{K=B_1} = \left( \frac{\partial C}{\partial K} \right)_{K=B_1} + \frac{e^{-r\tau} \Phi(d_-)}{\nu(B(t), K, r, \tau, \sigma_{impl})} - \frac{\Phi(d_-)}{\nu(B(t), K, r, \tau, \sigma_{impl})} < 0,
\]

(A6)

and a similar formula for \( \left( \frac{\partial \sigma_{impl}}{\partial K} \right)_{K=B_1} \). Taking the difference of these two formulas, we obtain (9).

Next, set \( q(t) = 1 \) and \( T_e = T \). Then \( \tau_e = \tau \), and

\[
\left( \frac{\partial \sigma_{impl}}{\partial K} \right)_{K=B_1} = \frac{e^{-r\tau} \nu(B, K, r, \tau, \sigma_{impl})}{\Phi(d_-)} \left( -1 + \Phi(d_-) \right) < 0,
\]

(A7)

\[
\left( \frac{\partial \sigma_{impl}}{\partial K} \right)_{K=B_1} = \frac{e^{-r\tau} \nu(B, K, r, \tau, \sigma_{impl})}{\Phi(d_-)} \left( \Phi(d_-) \right) > 0.
\]

By continuity, we obtain that the two inequalities above also hold when \( q(t) \) is sufficiently close to 1 and \( T_e \) is sufficiently close to \( T \).

Proof of Corollary 2. One simply uses Itô calculus to differentiate equation (6).

Proof of Proposition 2. We start with the general case when \( B_1 \) is stochastic. The formula (4) for the stock price \( B(t) \) does not depend on the expiration date \( T \). In particular, we have \( B(T) = q(T)B_1(T) + (1-q(T))B_2(T) \). If we denote by

\[
a = q(T)B_1(T) - K, \quad b = 1 - q(T), \quad k = \frac{K - q(T)B_1(T)}{1 - q(T)} = -\frac{a}{b},
\]

(A8)

we can write \( B(T) - K = b(B_2(T) - k) \). Note that each of the variables \( a, b \) and \( k \) is...
which implies $k > 0$ in (20), and (ii) the condition $T \leq B_\epsilon(T)^T$ expires at $T < T_e$. Its payoff at $T$ is

$$C^{K,T}(T) = (B(T) - K)_{+} = b(B_2(T) - k)_{+}.$$  \hfill (A9)

Note that $B(T) > K$ is equivalent to $B_2(T) > k$. There are two cases: $k \leq 0$ and $k > 0$. Then, the price of the call at $t < T$ is

$$C(t) = E_t^Q\left[e^{-r(T-t)} b (B_2(T) - k)_{+}\right] = E_t^Q\left[b E_t^Q\left(e^{-r(T-t)}(B_2(T) - k)_{+}\right)\right]$$

$$= P(k \leq 0) \cdot E_t^Q\left(b B_2(t) - e^{-r(T-t)} b k \mid k \leq 0\right)$$

$$+ P(k > 0) \cdot E_t^Q\left(b C_{BS}(B_2(t), k, r, T - t, \sigma_2) \mid k > 0\right)$$

$$= P(k \leq 0) \cdot E_t^Q\left(a e^{-r(T-t)} + b B_2(t) \mid k \leq 0\right)$$

$$+ P(k > 0) \cdot E_t^Q\left(a e^{-r(T-t)} \Phi\left(d_-(B_2(t), k, r, T - t, \sigma_2)\right) + b B_2(t) \Phi\left(d_+(B_2(t), k, r, T - t, \sigma_2)\right) \mid k > 0\right).$$ \hfill (A10)

This proves equation (22) if we show that (i) $q(T) = \Phi\left(\frac{\sqrt{T_e - t} \Phi^{-1}(q(t)) + \sqrt{T - t} \varepsilon_3}{\sqrt{T_e - T}}\right)$ is as in (20), and (ii) the condition $k > 0$ is equivalent to $\varepsilon_3 < \varepsilon$ where $\varepsilon$ is given by (21).

Recall that $q(t)$ is the risk neutral probability associated to a digital option that pays 1 at $T_e$ if a log-normal process $B_3$ is above a level $K_3$, or pays 0 otherwise. Then, $B_3(T_e) = B_3(t) \exp\left((r - \frac{\sigma_3^2}{2})(T_e - t) + \sigma_3 \sqrt{T_e - t} \varepsilon_3\right)$, where $\varepsilon_3 \sim \mathcal{N}(0, 1)$ has a standard normal distribution. The price of the digital option at $t$ is $p_m(t) = e^{-r(T_e-t)} \Phi\left(\frac{\ln\left(\frac{B_3(t)}{K_3}\right) + (r - \frac{\sigma_3^2}{2})(T_e - t)}{\sigma_3 \sqrt{T_e - T}}\right)$, which implies

$$q(T) = \Phi\left(\frac{\ln\left(\frac{B_3(T)}{K_3}\right) + (r - \frac{\sigma_3^2}{2})(T_e - T)}{\sigma_3 \sqrt{T_e - T}}\right)$$

$$= \Phi\left(\frac{\ln\left(\frac{B_3(t)}{K_3}\right) + (r - \frac{\sigma_3^2}{2})(T_e - t) + \sigma_3 \sqrt{T - t} \varepsilon_3}{\sigma_3 \sqrt{T_e - T}}\right)$$ \hfill (A11)

$$= \Phi\left(\frac{\sqrt{T_e - t} \Phi^{-1}(q(t)) + \sqrt{T - t} \varepsilon_3}{\sqrt{T_e - T}}\right).$$

This proves (20). The condition $k > 0$ is equivalent to $q(T) > \frac{K}{B_2(T)}$, which from
equation \((A11)\) is equivalent to \(\varepsilon_3 < \bar{\varepsilon}\), where
\[
\bar{\varepsilon} = \begin{cases} 
+\infty, & \text{if } K \geq B_1(T) \\
\sqrt{\frac{-e^{-r(T-t)}}{\sqrt{T-t}}} \Phi^{-1}\left(\frac{K}{B_1(T)}\right) - \sqrt{\frac{-e^{-r(T-t)}}{\sqrt{T-t}}} \Phi^{-1}(q(t)), & \text{if } K < B_1(T).
\end{cases}
\]
\[(A12)\]
which is the same as in \((21)\).

**Proof of Proposition 3.** The only non-trivial part of proving \((24)\) is the computation of
\[
E_t^Q(q(T_e)B_2(T_e)) = E_t^Q(1_{B_3(T_e) \geq K_3}B_2(T_e)),
\]
\[(A13)\]
where \(1_{B_3(T_e) \geq K_3}\) is the indicator function which is one if \(B_3(T_e) \geq K_3\), or zero otherwise.

From \((19)\) we get
\[
\ln \left(\frac{B_3(T_e)}{K_3}\right) = \ln \left(\frac{B_3(t)}{K_3}\right) + \left(\frac{r - \sigma_3^2}{2}\right)(T_e - t) + \sigma_3 \sqrt{T_e - t} \varepsilon_3,
\]
which implies that the condition \(B_3(T_e) \geq K_3\) is equivalent to \(\varepsilon_3 \geq -\Phi^{-1}(q(t))\). From \((19)\) we also write \(B_2(T_e) = a_2 e^{b_2 \varepsilon_2}\) for \(a_2 = B_2(t)\exp\left((r - \frac{\sigma_2^2}{2})(T_e - t)\right)\) and \(b_2 = \sigma_2 \sqrt{T_e - t}\). But \(\varepsilon_2\) and \(\varepsilon_3\) are bivariately normal with correlation \(\rho\). Hence, if we define \(\bar{\varepsilon}_3 = -\Phi^{-1}(q(t))\), we compute
\[
E_t^Q(1_{B_3(T_e) \geq K_3}B_2(T_e)) = a_2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{b_2 \varepsilon_2} f_{\rho}(\varepsilon_2, \varepsilon_3) \, d\varepsilon_2 \, d\varepsilon_3
\]
\[
= a_2 \int_{-\infty}^{+\infty} e^{b_2^2/2} \phi(\varepsilon_3 - \rho b_2) \, d\varepsilon_3
\]
\[(A14)\]
But \(a_2 e^{b_2^2/2} = B_2(t) e^{r(T_e-t)}\), and \(-\bar{\varepsilon}_3 = \Phi^{-1}(q(t))\), which implies
\[
E_t^Q(q(T_e)B_2(T_e)) = B_2(t) e^{r(T_e-t)} \Phi(\Phi^{-1}(q(t)) + \rho \sigma_2 \sqrt{T_e - t}).
\]
The rest of \((24)\) is straightforward to prove.

Similarly, the only non-trivial part of proving \((25)\) is the computation of
\[
E_t^Q\left[e^{-r(T-t)}q(T)(B_2(T) - K)^+\right] = E_t^Q\left[q(T_e) e^{-r(T_e-t)} E_T^Q\left(e^{-(T-T_e)}(B_2(T) - K)^+\right)\right]
\]
\[
= E_t^Q\left[1_{B_3(T_e) \geq K_3} e^{-r(T_e-t)} C_{BS}(B_2(T_e), T - T_e)\right],
\]
\[(A15)\]
where \(C_{BS}(S, T - t)\) is the Black–Scholes formula \((A5)\) with arguments \(K, r\) and \(\sigma_2\) omitted. Here we can no longer integrate along \(\varepsilon_2\) as before, because the function \(C_{BS}\)
is too complicated. Nevertheless, we integrate along $\varepsilon_3$, using the formula
\[
\int_{\varepsilon_3}^{+\infty} f_\rho(\varepsilon_2, \varepsilon_3) \, d\varepsilon_3 = \phi(\varepsilon_2) \Phi \left( \frac{\rho \varepsilon_2 - \overline{\varepsilon}_3}{\sqrt{1 - \rho^2}} \right). \tag{A16}
\]
Substituting (A16) into (A15), we get the same integral as in (25). The rest of the formula (25) is straightforward to prove.

\[\square\]

**Appendix B. MCMC Procedure for Cash Mergers**

In this section we describe a Markov Chain Monte Carlo (MCMC) method based on the state space representation of our model. The goal is to use the time series of observed stock prices and prices of various call options on the target companies, and estimate the time series of the two latent variables: the success probability of the merger $q(t)$, and the fallback price of the target $B_2(t)$.

In order to simplify the presentation, we start with specifications for $q$ and $B_2$ that are slightly different than the ones used in the empirical study.\(^{40}\) Define the state variables $X_1$ and $X_2$ as Itô processes with constant drift and volatility:
\[
dX_i(t) = \mu_i dt + \sigma_i dW_i(t), \quad i = 1, 2, \tag{B1}
\]
where $W_1(t)$ and $W_2(t)$ are independent standard Brownian motions. The latent variables $q$ and $B_2$ are related to the state variables by the following equations:
\[
q(t) = \frac{e^{X_1(t)}}{1 + e^{X_1(t)}}, \quad B_2(t) = e^{X_2(t)}. \tag{B2}
\]
The latent variables and the observed variables are connected by the observation equation, which puts together equations (14) and (15):
\[
B(t) = q(t)B_1 e^{-r(T_e-t)} + (1 - q(t))B_2(t) + \varepsilon_B(t),
\]
\[
C(t) = q(t)(B_1 - K)_+ e^{-r(T_e-t)} + (1 - q(t))C_{BS}(B_2(t), K, T - t) + \varepsilon_C(t), \tag{B3}
\]
\(^{40}\)Using Itô calculus, one sees that the specifications of $q$ and $B_2$ given here in equation (B2) differ from the ones used in the empirical part in equations (11) and (12) only up to a drift term.
where \( C_{BS}(S,K,T-t) \) is the Black–Scholes formula \((A5)\) with arguments \( r \) and \( \sigma_2 \) omitted. The errors \( \varepsilon_B(t) \) and \( \varepsilon_C(t) \) are IID normal with zero mean, and independent from each other. If more than one call option is employed in the estimation process, \( C(t) \) is multi-dimensional.

To simplify notation, we rename the observed variables: \( Y_B = B \), and \( Y_C = C \). The state variables are collected under \( X = [X_1,X_2]^T \), and the observed variables are collected under \( Y = [Y_B,Y_C]^T \). (The superscript \( T \) after a vector indicates transposition.)

There are other observed parameters: the effective date \( (T_e) \), the interest rate \( (r) \), the cash offer \( (B_1) \), and the strike prices \( (K) \) and maturities \( (T) \) of various call options on the company \( B \).

The vector of latent parameters is \( \theta = [\mu_1,\mu_2,\sigma_1,\sigma_2]^T \). The observation equations \((14)\) and \((15)\) can be rewritten as \( Y = f(X,\theta) + \varepsilon \), where \( \varepsilon = [\varepsilon_B,\varepsilon_C]^T \) is the vector of model errors. The diagonal matrix of model error variances, \( \Sigma_\varepsilon = \text{diag}(\sigma_{\varepsilon B}^2,\sigma_{\varepsilon C}^2) \) is called the matrix of hyperparameters.

The Markov Chain Monte Carlo (MCMC) method provides a way to sample from the posterior distribution with density \( p(\theta,X,\Sigma_\varepsilon|Y) \), and then estimate the parameters \( \theta \), the state variables \( X \), and the hyperparameters \( \Sigma_\varepsilon \). Bayes’ Theorem says that the posterior density is proportional to the likelihood times the prior density. In our case, \( p(X,\Sigma_\varepsilon,\theta|Y) \propto p(Y|X,\Sigma_\varepsilon,\theta) \cdot p(X,\Sigma_\varepsilon,\theta) = p(Y|X,\Sigma_\varepsilon,\theta) \cdot p(X|\theta) \cdot p(\Sigma_\varepsilon) \cdot p(\theta) \). On the right hand side, the first term in the product is the likelihood for the observation equation; the second term is the likelihood for the state equation; and the third and fourth terms are the prior densities of the hyperparameters \( \Sigma_\varepsilon \) and the parameters \( \theta \).

We obtain

\[
p(Y|X,\Sigma_\varepsilon,\theta) = \prod_{t=1}^{T_e} \phi(Y(t)|f(X(t),\theta),\Sigma_\varepsilon), \quad p(X|\theta) = p(X(1)|\theta) \cdot \prod_{t=2}^{T_e} \phi(Z(t)|\mu,\Sigma_X),
\]

where \( Z_i(t) = X_i(t) - X_i(t-1), \mu = [\mu_1,\mu_2]^T, \Sigma_X = \text{diag}(\sigma_1^2,\sigma_2^2), \) and \( \phi(x|\mu,\Sigma) = (2\pi)^{-n/2}|\Sigma|^{-1/2} \exp\left(-\frac{1}{2}(x-\mu)^\top \Sigma^{-1}(x-\mu)\right) \) the \( n \)-dimensional multivariate normal density with mean \( \mu \) and covariance matrix \( \Sigma \).

We now describe the MCMC algorithm.

**STEP 0.** Initialize \( \theta^{(1)}, X^{(1)}, \Sigma_\varepsilon^{(1)} \). Fix a number of iterations \( M \). Then for each
Recall that \( \theta \) the following updates:

From the formula, since it does not contain those parameters. In that case, we have

\[
p(t) \propto \prod_{i=1}^{T_e} \phi(Y(t)|f(X(t), \theta), \Sigma) .
\]  \hspace{1cm} (B5)

This implies that \( \sigma_{i,j}^{(i+1)} \), \( j = B, C \), is sampled from an inverted gamma-2 distribution, \( IG_2(s, \nu) \), where \( s = \sum_{t=1}^{T_e} (Y_j(t) - f_j(X(t), \theta))^2 \) and \( \nu = T_e - 1 \). The inverted gamma-2 distribution \( IG_2(s, \nu) \) has log-density \( \ln p_{IG_2}(x) = -\frac{\nu+1}{2} \ln(x) - \frac{s}{2x} \). One could also use a conjugate prior for \( \Sigma \), which is also an inverted gamma-2 distribution.

### STEP 1.

Update \( \Sigma^{(i+1)}_\varepsilon \) from \( p(\Sigma_\varepsilon|\theta^{(i)}, X^{(i)}, Y) \). Note that with a flat prior for \( \Sigma \),

\[
p(\Sigma_\varepsilon|\theta^{(i)}, X^{(i)}, Y) \propto \prod_{t=1}^{T_e} \phi(Y(t)|f(X(t), \theta), \Sigma).
\]

STEP 2. Update \( X^{(i+1)} \) from \( p(X|\theta^{(i)}, \Sigma^{(i+1)}_\varepsilon, Y) \). To simplify notation, denote by \( \theta = \theta^{(i)} \), and \( \Sigma_\varepsilon = \Sigma^{(i+1)}_\varepsilon \). Notice that \( p(X|\theta, \Sigma_\varepsilon, Y) \propto p(Y|\theta, \Sigma_\varepsilon, X) \cdot p(X|\theta) \), assuming flat priors for \( X \). Then, if \( t = 2, \ldots, T_e - 1 \),

\[
p(X(t)|\theta, \Sigma_\varepsilon, Y) \propto \phi(Y(t)|f(X(t), \theta), \Sigma_\varepsilon)
\cdot \phi(X(t) - X^{(i+1)}(t - 1)|\mu, \Sigma_X) \hspace{1cm} (B6)
\cdot \phi(X^{(i)}(t) - X(t)|\mu, \Sigma_X).
\]

If \( t = 1 \), replace the second term in the product with \( p(X(1)|\theta) \); and if \( t = T_e \), drop the third term out of the product. This is a non-standard density, so we perform the Metropolis–Hastings algorithm to sample from this distribution. This algorithm is described at the end of next step.

### STEP 3.

Update \( \theta^{(i+1)} \) from \( p(\theta|X^{(i+1)}, \Sigma^{(i+1)}_\varepsilon, Y) \). To simplify notation, denote by \( X = X^{(i+1)} \), and \( \Sigma_\varepsilon = \Sigma^{(i+1)}_\varepsilon \). Assuming a flat prior for \( \theta \), \( p(\theta|X, \Sigma_\varepsilon, Y) \propto p(Y|\theta, \Sigma_\varepsilon, X) \cdot p(X|\theta) \). Then, if we assume that \( X(1) \) does not depend on \( \theta \),

\[
p(\theta|X, \Sigma_\varepsilon, Y) \propto \prod_{t=1}^{T_e} \phi(Y(t)|f(X(t), \theta), \Sigma_\varepsilon) \cdot \prod_{t=2}^{T_e} \phi(X(t) - X(t - 1)|\mu, \Sigma_X). \hspace{1cm} (B7)
\]

Recall that \( \theta = [\mu_1, \mu_2, \sigma_1, \sigma_2]^T \). For \( \mu_1, \mu_2, \) and \( \sigma_1, \) we can drop the first product from the formula, since it does not contain those parameters. In that case, we have the following updates:

\[
\mu_k^{(i+1)} \sim \Phi \left( \frac{1}{T_e - 1} \sum_{t=2}^{T_e} (X_k(t) - X_k(t - 1))^2, \frac{(\sigma_k^{(i)})^2}{T_e - 1} \right), \; k = 1, 2;
\]

\[
\sigma_k^{(i+1)} \sim \sigma_k^{(i)} \Phi \left( \frac{1}{T_e - 1} \sum_{t=2}^{T_e} (X_k(t) - X_k(t - 1))^2, \frac{(\sigma_k^{(i)})^2}{T_e - 1} \right), \; k = 1, 2.
\]
\((\sigma_{1}^{(i+1)})^2 \sim IG_2 \left( \sum_{t=2}^{T_e} (X_1(t) - X_1(t-1) - \mu_{1}^{(i+1)})^2, T_e - 2 \right)\). For the other parameters the density is non-standard, so we need to perform the Metropolis–Hastings algorithm.

**METROPOLIS–HASTINGS.** The goal of this algorithm is to draw from a given density \(p(x)\). Start with an element \(X_0\), which is given to us from the beginning. (E.g., in the MCMC case, \(X_0\) is the value of a parameter \(\theta^{(i)}\), while \(X\) is the updated value \(\theta^{(i+1)}\)). Take another density \(q(x)\), from which we know how to draw a random element. Initialize \(X_{CURR} = X_0\). The Metropolis–Hastings algorithm consists of the following steps:

1. Draw \(X_{PROP} \sim q(x \mid X_{CURR})\) (this is the “proposed” \(X\)).

2. Compute \(\alpha = \min \left( \frac{p(X_{PROP})}{p(X_{CURR})}, \frac{q(X_{CURR} \mid X_{PROP})}{q(X_{PROP} \mid X_{CURR})} \right), 1 \).

3. Draw \(u \sim U[0, 1]\) (the uniform distribution on [0, 1]). Then define \(X^{(i+1)}\) by: if \(u < \alpha\), \(X^{(i+1)} = X_{PROP}\) (“accept”); if \(u \geq \alpha\), \(X^{(i+1)} = X_{CURR}\) (“reject”).

Typically, we use the “Random-Walk Metropolis–Hastings” version, for which \(q(y \mid x) = \phi(x \mid 0, a^2)\), for some positive value of \(a\). Equivalently, \(X_{PROP} = X_{CURR} + e\), where \(e \sim \mathcal{N}(0, a^2)\).

In our empirical study, we choose the number of iterations to be \(M = 400,000\) and we observe that the algorithm typically converges (i.e., the estimated posterior density appears stationary) after an initial “burn-out” period of about 200,000 iterations. With these numbers, it takes our algorithm on average about one day per firm to finish at our current computing speed. Since we use the Metropolis–Hastings algorithm described above, we follow standard procedure and require that the average acceptance ratios are between 0.04 and 0.96; otherwise, we modify the value of the random walk parameter \(a\) until we obtain acceptance ratio in this interval. If this step fails as well, we exclude the company from the sample. A total of 2 companies have been excluded for this reason.

**REFERENCES**


Figure 4: Success Probability Estimates for Ten Selected Cash Mergers. Using the methodology in the paper, this figure plots the success probability for the subsample of ten cash mergers described in Table 3. The deals corresponding to target tickers BUD, AWE, TXU, AT, FDC succeeded, while those for SLM, MCIC, UCL, HET, AVP failed. The dash-dotted lines represent the 5% and 95% error bands around the estimated median values. The estimates are obtained using the call options on the target company with the earliest expiration date after the effective date of the merger.
Figure 5: Comparison of Observed and Theoretical Volatility Smiles for AWE. From the sample of ten large cash merger deals in Table 3 we select the deal with the largest offer premium (75.44%). The deal has target company AT&T Wireless, with ticker AWE. The figure plots the observed and theoretical volatility smiles of call options traded on AWE, for equally spaced trading days during the merger deal. The option expiration date is the earliest date after the effective date of the merger. On the x-axis we plot the ratio $K/B_1$ (call strike price $K$ to merger offer price $B_1$). On the y-axis we plot (i) the Black–Scholes implied volatility for the observed call price, using either a star or a dot: a star for an option with positive trading volume, or a dot for an option with zero volume; (ii) the Black–Scholes implied volatility for the theoretical call price (based on the model in this paper), using a continuous solid line. The parameters used to compute the theoretical price are estimated by using each day only the option with the highest trading volume on that day.
Table 1: Data Description. This table reports summary statistics for our sample of cash mergers from January 1996 to December 2014 for which there are option prices quoted on the target company. Included are the mean, standard deviation, and various percentiles for (i) the number of trading days between deal announcement and deal conclusion (Deal Duration); (ii) the percentage difference between the offer price per share and the share price for the target company one day before the deal was announced (Offer Premium); (iii) the percentage of trading days for which there is at least one option with positive trading volume (Frac. Days Calls Traded); (iv) the average number of call option contracts (1 contract = 100 options) traded on the target company (Ave. Call Volume); (v) the number of times the offer price changed during the merger deal (Offer Price Changes); (vi) the number of times the effective date of the merger was changed during the merger deal (Effec. Date Changes). \( N \) represents the number of firms with non-missing estimates.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>StDev</th>
<th>Min</th>
<th>5%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>95%</th>
<th>Max</th>
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<td>812</td>
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<td>Offer Premium (%)</td>
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</tr>
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<td>68.29</td>
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<td>1</td>
<td>4</td>
<td>21</td>
<td>812</td>
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Table 2: Bid-Ask Spreads of Call Options. For target company $i$ in our sample of cash mergers, we report statistics for $\mu_i$, the average absolute bid-ask spread (Panel A) or relative bid-ask spread (Panel B). This average is computed over several categories of options that depend on the option’s moneyness $m = K/S$ ($K$ is the strike price and $S$ is the underlying stock price): all calls; deep-in-the-money (Deep-ITM) calls, with $m < 0.9$; in-the-money (ITM) calls, with $m \in [0.9, 0.95]$; near-in-the-money (Near-ITM) calls, with $m \in [0.95, 1]$; near-out-the-money (Near-OTM) calls, with $m \in [1, 1.05]$; out-of-the-money (OTM) calls, with $m \in (1.05, 1.1]$; and deep-out-of-the-money (Deep-OTM) calls if $m > 1.1$. Entries with zero bid-ask spread or zero bid price are considered as missing. $N$ represents the number of target firms with at least one non-missing estimate.

Panel A: Absolute Bid-Ask Spread ($$)

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<th>StDev</th>
<th>Min</th>
<th>5%</th>
<th>25%</th>
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Panel B: Relative Bid-Ask Spread (%)

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<td>5.37</td>
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<td>93.15</td>
<td>119.23</td>
<td>154.81</td>
<td>196.04</td>
<td>487</td>
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</table>
Table 3: Ten Selected Cash Mergers. This table reports information on ten large deals sorted on offer value (offer price per share times the target’s number of shares outstanding). From our sample of cash mergers with options traded on the target company, we select the five largest deals that succeeded, and the five largest deals that failed. Panel A reports the names of the acquirer and target company, the ticker of the target company, and the offer value in billion U.S. dollars. Panel B reports the target’s ticker, the deal announcement date, the date when the deal succeeded or failed, the target’s stock price one day before the announcement $B(−1)$, the offer price $B_1$, and the offer premium which is the percentage change $\frac{B_1- B(−1)}{B(−1)}$.

Panel A: List of Deals

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<tr>
<th>Acquirer Name</th>
<th>Target Name</th>
<th>Tgt.Ticker</th>
<th>Offer Value ($ bn)</th>
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</thead>
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<tr>
<td>InBev NV</td>
<td>Anheuser-Busch Cos Inc</td>
<td>BUD</td>
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</tr>
<tr>
<td>Cingular Wireless LLC</td>
<td>AT&amp;T Wireless Services Inc</td>
<td>AWE</td>
<td>40.72</td>
</tr>
<tr>
<td>TXU Corp SPV</td>
<td>TXU Corp</td>
<td>TXU</td>
<td>31.80</td>
</tr>
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<td>Atlantis Holdings LLC</td>
<td>Alltel Corp</td>
<td>AT</td>
<td>25.76</td>
</tr>
<tr>
<td>Kohlberg Kravis Roberts &amp; Co</td>
<td>First Data Corp</td>
<td>FDC</td>
<td>25.60</td>
</tr>
<tr>
<td>Investor Group</td>
<td>SLM Corp</td>
<td>SLM</td>
<td>24.53</td>
</tr>
<tr>
<td>GTE Corp</td>
<td>MCI Communications Corp</td>
<td>MCIC</td>
<td>22.24</td>
</tr>
<tr>
<td>China National Offshore Oil</td>
<td>Unocal Corp</td>
<td>UCL</td>
<td>18.20</td>
</tr>
<tr>
<td>Penn National Gaming Inc</td>
<td>Harrahs Entertainment Inc</td>
<td>HET</td>
<td>16.17</td>
</tr>
<tr>
<td>Coty US Inc</td>
<td>Avon Products Inc</td>
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Panel B: Deal Information

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<th>Date Announced</th>
<th>Date Ended</th>
<th>Deal Status</th>
<th>Price 1 Day. Before Ann.</th>
<th>Offer Price ($)</th>
<th>Offer Premium (%)</th>
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<td>11/18/2008</td>
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<td>Completed</td>
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<td>69.25</td>
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<td>AT</td>
<td>5/20/2007</td>
<td>11/16/2007</td>
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<td>58.19</td>
<td>71.5</td>
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<td>60</td>
<td>47.24</td>
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<td>8/2/2005</td>
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<td>67</td>
<td>51.11</td>
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<td>12/19/2006</td>
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<td>87</td>
<td>10.88</td>
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<td>5/14/2012</td>
<td>Withdrawn</td>
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<td>24.75</td>
<td>26.51</td>
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Table 4: Relative Pricing Errors for the Stock Price. For target company \( i \) in our sample of cash mergers, we report statistics for \( \mu_i \), the time series average of the relative stock pricing error. On trading day \( t \), the relative pricing error is \( \left( S^{\text{BMR}}_i(t) - S_i(t) \right) / S_i(t) \), where \( S_i(t) \) is the target company’s observed stock price, and \( S^{\text{BMR}}_i(t) \) is the theoretical stock price (based on the model in this paper). The parameters used to compute the theoretical price are estimated by using each day only the option with the highest trading volume on that day. We winsorize the pricing error at 100%: for one merger deal the average pricing error is 130.89%. \( N \) represents the number of firms with non-missing estimates.

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Mean</th>
<th>StDev</th>
<th>Min</th>
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<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>95%</th>
<th>Max</th>
<th>N</th>
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</thead>
<tbody>
<tr>
<td>Pricing Error (%)</td>
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<td>0.001</td>
<td>0.003</td>
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<td>0.122</td>
<td>0.653</td>
<td>3.511</td>
<td>811</td>
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</table>
Table 5: Pricing Errors for Call Options. This table reports absolute and relative implied volatility errors for call options traded on the target company in our sample of cash mergers. The theoretical implied volatilities $\sigma_{\text{impl}}$ are computed according to two models: BMR (the model in this paper) and BS (the Black–Scholes model). For target company $i$ we report statistics for $\mu_i$, the average absolute or relative pricing error for call options traded on $i$. This average is computed over several categories of options that depend on the option’s moneyness $m = K/S$ ($K$ is the strike price and $S$ is the underlying stock price): all calls; deep-in-the-money (Deep-ITM) calls, with $m < 0.9$; in-the-money (ITM) calls, with $m \in [0.9, 0.95)$; near-in-the-money (Near-ITM) calls, with $m \in [0.95, 1]$; near-out-the-money (Near-OTM) calls, with $m \in [1, 1.05]$; out-of-the-money (OTM) calls, with $m \in (1.05, 1.1]$; and deep-out-of-the-money (Deep-OTM) calls if $m > 1.1$. On each day $t$ we require the first category (all calls) to have at least 6 options, and the other categories (calls of a given moneyness) to have at least 2 (quoted) options, otherwise we declare as missing the pricing errors on that day. The absolute error is the difference $|\sigma_{\text{theory}}^{\text{impl}}(t) - \sigma_{\text{obs}}^{\text{impl}}(t)|$, and the relative error is $|\sigma_{\text{theory}}^{\text{impl}}(t) - \sigma_{\text{impl}}^{\text{obs}}(t)/\sigma_{\text{impl}}^{\text{obs}}(t)|$, where $\sigma_{\text{impl}}^{\text{BMR}}(t)$ is the Black–Scholes implied volatility corresponding to the theoretical underlying and call prices computed using the BMR model; $\sigma_{\text{impl}}^{\text{obs}}(t)$ is the Black–Scholes implied volatility corresponding to the observed underlying and call prices; and $\sigma_{\text{impl}}^{\text{BS}}(t)$ is constant and equal to the average over $t$ of at-the-money Black–Scholes implied volatility corresponding to the observed underlying and call prices. The parameters used to compute the BMR theoretical price are estimated by using each day only the option with the highest trading volume on that day. $N$ represents the number of firms with non-missing estimates.
Panel A: Absolute Implied Volatility Errors ($)

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<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>95%</th>
<th>Max</th>
<th>N</th>
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</thead>
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<tr>
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<td>0.10</td>
<td>0.02</td>
<td>0.05</td>
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<td>0.13</td>
<td>0.21</td>
<td>0.36</td>
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<tr>
<td></td>
<td>BS</td>
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<td>0.15</td>
<td>0.05</td>
<td>0.10</td>
<td>0.18</td>
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<td>0.83</td>
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Panel B: Relative Implied Volatility Errors (%)

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<th>95%</th>
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<td>74.77</td>
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<td>364.15</td>
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Table 6: Ratio of Absolute Call Option Pricing Errors to Half Spread. For target company $i$ in our sample of cash mergers, we report statistics for $\mu_i$, the average ratio of the absolute option pricing error to the option bid-ask spread. This average is computed over several categories of options that depend on the option’s moneyness $m = K/S$ ($K$ is the strike price and $S$ is the underlying stock price): all calls; deep-in-the-money (Deep-ITM) calls, with $m < 0.9$; in-the-money (ITM) calls, with $m \in [0.9, 0.95]$; near-in-the-money (Near-ITM) calls, with $m \in [0.95, 1]$; near-out-the-money (Near-OTM) calls, with $m \in [1, 1.05]$; out-of-the-money (OTM) calls, with $m \in (1.05, 1]$; and deep-out-of-the-money (Deep-OTM) calls if $m > 1.1$. The absolute pricing error is the difference $|C_{BMR}(t) - C_{obs}(t)|$ between theoretical and observed call prices. The parameters used to compute the BMR theoretical price are estimated by using each day only the option with the highest trading volume on that day. $N$ represents the number of firms with non-missing estimates.

<table>
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<tr>
<th>Selection</th>
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<th>StDev</th>
<th>Min</th>
<th>5%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>95%</th>
<th>Max</th>
<th>$N$</th>
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<td>0.01</td>
<td>0.15</td>
<td>0.33</td>
<td>0.48</td>
<td>0.70</td>
<td>2.08</td>
<td>16.28</td>
<td>567</td>
</tr>
<tr>
<td>OTM Calls</td>
<td>0.65</td>
<td>0.77</td>
<td>0.01</td>
<td>0.19</td>
<td>0.37</td>
<td>0.50</td>
<td>0.65</td>
<td>1.78</td>
<td>12.51</td>
<td>535</td>
</tr>
<tr>
<td>Deep-OTM Calls</td>
<td>0.55</td>
<td>0.31</td>
<td>0.13</td>
<td>0.34</td>
<td>0.46</td>
<td>0.50</td>
<td>0.50</td>
<td>1.02</td>
<td>3.81</td>
<td>709</td>
</tr>
</tbody>
</table>
Table 7: The Call Price Kink and the Success Probability. For target company $i$ in our sample of cash mergers, consider the call options traded on date $t$ which expire at the earliest date after the effective date of the merger. Let $C_{i}^{\text{kink}}(t)$ be the call price kink, i.e., the difference between the right slope and left slope of option prices corresponding to the strike prices $K$ nearest to the merger offer price. Let $q_{i}(t)$ be the success probability estimated at $t$ using only one option per day (with the highest trading volume). We write the same variables with a “tilde” ($\tilde{C}_{i}^{\text{kink}}(t)$ and $\tilde{q}_{i}(t)$) by truncating their values to be in the interval $(0,1)$ and inverting them via the standard normal CDF function. The table reports the results of panel regressions of the call price kink on the success probability. The $t$-statistics are in parentheses, and standard errors are clustered by firm.

<table>
<thead>
<tr>
<th></th>
<th>$C^{\text{kink}}$</th>
<th>$\tilde{C}^{\text{kink}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>const.</td>
<td>0.230</td>
<td>0.291</td>
</tr>
<tr>
<td></td>
<td>(6.76)</td>
<td>(5.32)</td>
</tr>
<tr>
<td>$q_{BMR}$</td>
<td>0.705</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(13.69)</td>
<td></td>
</tr>
<tr>
<td>$\tilde{q}_{BMR}$</td>
<td></td>
<td>0.954</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(11.68)</td>
</tr>
<tr>
<td>$N$</td>
<td>41,354</td>
<td>41,354</td>
</tr>
<tr>
<td>Clusters</td>
<td>682</td>
<td>682</td>
</tr>
<tr>
<td>$R^{2}$</td>
<td>8.85%</td>
<td>9.12%</td>
</tr>
</tbody>
</table>
Table 8: The Success Probability as a Predictor of Deal Outcome. For target company \( i \) in our sample of cash mergers, let \( q_{i}^{\text{BMR}}(t) \) be the success probability estimated at date \( t \) using only one option per day (with the highest trading volume). Also, let \( q_{i}^{\text{naive}}(t) \) be the "naive" success probability, computed as the ratio \( \frac{S_{i}(t) - S_{i}(-1)}{S_{\text{offer}} - S_{i}(-1)} \), where \( S_{i}(t) \) is the current stock price, \( S_{i}(-1) \) is the stock price one day before the merger announcement, \( S_{\text{offer}} \) is the offer price, and the ratio is truncated to be in the interval \((0, 1)\). To create a panel, we consider 10 equally spaced days, \( t_{1}, \ldots, t_{10} \), throughout the life of the deal (between deal announcement and deal conclusion). We run probit regressions, where the dependent variable is the deal’s outcome (success or failure), and the independent variable is either \( q_{i}^{\text{BMR}} \) for regressions (1) and (3), or \( q_{i}^{\text{naive}} \) for regressions (2) and (4). Regressions (1) and (2) are carried for all time values \((t_{1} \text{ to } t_{10})\), while regressions (3) and (4) are carried for the second half of the deal \((t_{6} \text{ to } t_{10})\). The \( t \)-statistics are in parentheses, and are obtained from standard errors clustered by firm.

\[
\begin{array}{cccc}
\text{const.} & (1) & (2) & (3) \\
-0.937 & -0.557 & -1.450 & -1.056 \\
(-6.24) & (-3.72) & (-8.38) & (-6.13) \\
q_{i}^{\text{BMR}} & 4.069 & 5.131 \\
(13.04) & (13.79) \\
q_{i}^{\text{naive}} & 2.154 & 2.732 \\
(12.81) & (14.05) \\
N & 8,120 & 8,100 & 4,060 & 4,050 \\
Clusters & 812 & 810 & 812 & 810 \\
Pseudo-R^{2} & 32.33\% & 20.01\% & 46.20\% & 29.92\% \\
\end{array}
\]